Fuzzy Measures of Students' Mathematical Modelling Skills

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Abstract

This paper reports on an experimental study on mathematical modelling (MM) with subjects postgraduate students of the Faculty of Computer and Information Sciences of the Ain Shams University, Cairo, Egypt. The Voskoglou's fuzzy methods for the process of modelling developed in earlier papers were applied in this experiment. The results obtained are compared with corresponding results of students' of the Graduate Technological Educational Institute of Patras, Greece reported in earlier works of Vokoglou and some new useful conclusions are stated.

Keywords: *Fuzzy sets and logic, total possibilistic uncertainty, centroid defuzzification technique, mathematical modelling, students' assessment, computational thinking.*

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1. Introduction

The *systems' modelling* is a basic principle in engineering, in natural and in social sciences. When handling a problem concerning a system's operation (e.g. maximizing the productivity of an organization, minimizing the functional costs of a company, etc) a model is required to describe and represent the system's multiple views. The model is a simplified representation of the basic characteristics of the real system including only its entities and features under concern. In this sense, no model of a complex system could include all features and/or all entities belonging to the system. In fact, in this way the model's structure could become very complicated and therefore its use in practice could be very difficult and sometimes impossible. Therefore the construction of the model usually involves a deep abstracting process on identifying the system's dominant variables and the relationships governing them.

We recall that the main stages of the modelling process involve the *analysis* of the given problem, the *construction* of the model, the *solution*, the *validation (control)* of the model and the *implementation* of the final results to the real system. We recall also that the most important type of model in use is the *mathematical model*, where all variables are quantifiable. In this case the construction of the model is usually referred as *mathematization*. For more details see, for example, section 1 of [1].

A system can be viewed as a bounded transformation, i.e. as a process or a collection of processes that transforms inputs into outputs with the very broad meaning of the concept. For example, an output of a passengers' bus is the movement of people from departure to destination. Many of these processes are frequently characterized by a degree of vagueness and/or uncertainty. For example, during the processes of learning, of reasoning, of problem-solving, of modelling, etc, the human cognition utilizes in general concepts that are inherently graded and therefore fuzzy. On the other hand, from the teacher's point of view there usually exists an uncertainty about the degree of students' success in each of the stages of the corresponding didactic situation.

All the above gave to Prof. Voskoglou, the first of the authors of this paper, the impulsion to develop a fuzzy model for a more effective description of the modelling process (see [1] and [2]). In [2] Voskoglou used the *total possibilistic uncertainty* as a measure of a student group's modelling skills, while in [1] he used the *centroid defuzzificaion technique* for the same purpose. Also Voskoglou introduced in [3] techniques for the students' individual assessment Further, Voskoglou and Buckley developed in [4] a detailed account of *computational thinking*, which is a new problem solving method named for its extensive use of computer science techniques. The results of the classroom experiments presented in [1] and [4] suggest that the use of computers as a tool for problem solving enhances the students' abilities in solving real world problems involving mathematical modelling.

The two authors of the present paper discussed in detail the above experimental results and they decided to investigate further the situation together. This paper reports on a recent experimental study on this matter with subjects postgraduate students of the Faculty of Computer and Information Sciences of the Ain Shams University, Cairo, Egypt. The Voskoglou's fuzzy methods mentioned above were also applied in this case. The results obtained are compared with the corresponding results of students' of the Graduate Technological Educational Institute of Patras, Greece reported in [1] and [2] and some new useful conclusions are stated.

For general facts on fuzzy sets and on uncertainty theory we refer freely to the book [5].

2. The Fuzzy Model

The main ideas of Voskoglou's fuzzy model for the process of mathematical modelling (MM) developed in [1] and [2] are the following:

Let us consider a group of n students, $n\geq 2$, during the MM process. We denote by A_i , $i=1,2,3$, the stages of analysis/mathematization, solution and validation/implementation respectively, and by a, b, c, d, and e the linguistic labels of negligible, low, intermediate, high and complete success respectively at each of the A_i*'s*. The consideration of analysis/mathematization and of validation/implementation as join stages was done in order to make the model technically simpler.

Set $U=\{a, d, c, d, e\}$ and denote by n_{ia}, n_{ib}, n_{ic}, n_{id} *and* n_{ie} the numbers of students that have achieved negligible, low, high and complete success at state A_i respectively, $i=1,2,3$, In order to represent the A_i's as fuzzy subsets of U. we define the membership function m_{A_i} for all x in U by

$$m_{A_i}(x) = \begin{cases} 1, & if \quad \frac{4n}{5} < n_{ix} \leq n \\ 0,75, & if \quad \frac{3n}{5} < n_{ix} \leq \frac{4n}{5} \\ 0,5, & if \quad \frac{2n}{5} < n_{ix} \leq \frac{3n}{5} \\ 0,25, & if \quad \frac{n}{5} < n_{ix} \leq \frac{2n}{5} \\ 0, & if \quad 0 \leq n_{ix} \leq \frac{n}{5} \end{cases}$$

Then we can write $A_i = \{(x, m_{Ai}(x)): x \in U\}, i=1, 2, 3.$

A *student's profile* during the MM process is defined to be an ordered triple of the form $s = (x, y, z)$, where x, y and z are elements of U that denote the student's success at the stages A_1, A_2 and A_3 respectively. A profile s is said to be *well ordered* if x corresponds to a degree of success equal or greater than y and y corresponds to a degree of success equal or greater than z. The membership degree m_s of each profile s is defined to be the product $m_s = m_{A_1}(x).m_{A_2}(y).m_{A_3}(z)$ if s is well ordered, and *0* otherwise. In fact, if for example the profile *(b, a, c)* possessed a nonzero membership degree, how it could be possible for a modeller, who has failed in solving the model, to perform satisfactorily in validating/implementing it?

The rest of the model involves the calculation of the possibilities of all profiles by the well known formula $r_s = \frac{m_s}{\max\{m_s\}}$, where $max\{m_s\}$ denotes the maximal value of m_s, for all s in U^3. In other words r_s is the "relative membership degree" of s with respect to the membership degrees of the other profiles.

In this way we obtain a qualitative view of the students' performance during the learning process of a subject matter in the classroom. This is reinforced by Shackle [6] (and many others after him), who argues that human cognition can be formalized more adequately by possibility rather, than by probability theory. We recall the probability for fuzzy data is defined by $p_s = \frac{m_s}{\sum_{s \in U^3} m_s}$, which gives that $p_s \leq r_s$ for all s in U^3. This is compatible to the common logic, since whatever it is probable it is also possible, but whatever is possible need not be very probable.

A basic principle of the information theory states that the amount of information obtained by an action can be measured by the reduction of uncertainty that results from the action. Thus a measure of a student group's uncertainty can be also adopted as a measure of its performance. For example, the lower is a group's uncertainty after the MM process, which indicates a greater reduction of it during this process, the better is the group's performance.

In [2] Voskoglou used a student group's *total possibilistic uncertainty* (i.e. the sum of *strife* and *non specificity*) as a measure of its performance during the MM process. Other measures of uncertainty that are commonly used in fuzzy logic involve the *total probabilistic uncertainty***,** i.e. the classical *Shannon's entropy* expressed in terms of the Dempster-Shafer mathematical theory of evidence for use in a fuzzy environment (e.g. see [7]) and the *ambiguity* which is a generalization of the Shannon's entropy in possibility theory that captures both strife and non specificity (e.g. see [8]).

Another popular technique of producing a quantifiable result from fuzzy data (*defuzzification*) is the *centroid method*, in which the coordinates (x_c, y_c) of the centre of gravity of the graph of the membership function involved provide an alternative measure of a group's performance (e.g. see [9], [10], etc). We recall that in order to apply the centroid defuzzification technique one must correspond to each *x* of *U* an interval of values from a prefixed numerical distrbution. In our case (MM process) we characterize a student's performance as very low *(a)* if $x \in [0, 1)$, as low *(b)* if

$x \in [1, 2)$, as intermediate *(c)* if $x \in [2, 3)$, as high *(d)* if $x \in [3, 4)$ and as very high *(e)* if $x \in [4,5]$ respectively[1]. Then the graph of the corresponding membership function $y = m_{A_i}(x)$ is a bar graph consisting of five rectangles (e.g. see Figure 1 of [1]), whose sides on the x axis have length 1. The centroid method enables one to compare the student groups' performance at each stage of the MM process. In fact, according to the criterion developed in section 3 of [1], the greater the value of x_c , the better the group's performance at the corresponding stage. We recall that the value of x_c is calculated by the formula $x_c = \frac{1}{2}(\frac{y_1 + 3y_2 + 5y_3 + 7y_4 + 9y_5}{y_1 + y_2 + y_3 + y_4 + y_5})$, where $y = m_{A_i}(x)$, *i=1, 2, 3* [1].

The above two assessment approaches treat differently the idea of the students' performance and therefore the results obtained may differ to each other. In fact, in the first case the student group's uncertainty during the MM process is connected to its capacity in obtaining the relevant information. In other words, in this case we are looking for the *average group's performance*. On the other hand, in the case of the centroid technique the *weighted average* plays the main role, i.e. the results of the performance close to the ideal performance have much more weight than those close to the lower end. In other words, in this case we are mostly looking at the *quality* of the performance. It is argued that the combined application of these two approaches helps in finding the ideal profile of performance according to the user's personal criteria of goals and therefore to finally choosing the appropriate approach for measuring the results of his/her experiments.

[1] In practice this means that we characterize each student's performance with a numerical value (mark) instead of a linguistic label.

Voskoglou's fuzzy model for MM can be used also (in a simplified form) for the students' individual assessment. In fact, if $n=1$, then from the definition of the membership function m_{A_i} it becomes evident that in each A_i, i = 1, 2, 3, there exists a unique element x of U with membership degree 1, while all the others have membership degree 0. Consequently there exists a unique student profile s with $m_s= 1$, while all the others have membership degree 0. For example, if $m_{A_1}(d)=1,\ m_{A_2}(c)=1,\ m_{A_3}(b)=1$, then s = (d, c, b). In other words, each student is characterized in this case by a unique profile, which gives us the requested information about his/her performance. The above characterization defines in general a relationship of partial order among students' concerning their performance. For example, the profile (d, c, b) indicates a better performance than the profile *(c, b, b)*. On the contrary, in case of profiles (d, c, b) and (c, c, c) the student possessing the first one demonstrates a better performance at the stage of analysis/mathematization of the MM process, while the student possessing the second one demonstrates a better performance at the stage of validation/implementation.

A. Jones developed a fuzzy model to the field of Education involving several theoretical constructs related to assessment, amongst which is a technique for assessing the deviation of a student's knowledge with respect to the teacher's knowledge , which is taken as a reference (see [11], [12]). Here we shall present this technique, properly adapted with respect to Voskoglou's fuzzy model, as an alternative fuzzy method for the individual assessment of MM skills (see also [3])

Let $X= \{A_1, A_2, A_3\}$ be the set of the stages of MM process as they have been considered above. Then a fuzzy subset of X of the form

$\{(A_1, m(A_1)), (A_2, m(A_2)), (A_3, m(A_3)\}$can be assigned to each student , where the membership function m takes the values $0, 0.25, 0.5, 0.75, 1$ according to the level of the student's performance at the corresponding step. The teacher's fuzzy measurement is always equal to 1, which means that the fuzzy subset of X corresponding to the teacher is $\{(A_1, 1), (A_2, 1), (A_3, 1)$.

Then the *fuzzy deviation* of the student i with respect to the teacher is defined to be the fuzzy subset $D_i=\{(A_1,1-m(A_1)), (A_2, 1-m(A_2)), (A_3,1- m(A_3)\}$ of X. This assessment by reference to the teacher provides us with the ideal student as the one with nil deviation in all his/her components.

Notice that each deviation D_i corresponds uniquely to a student's profile s_i. For example, the deviation $D_i = \{(A_1,0.75), (A_2, 0.75), (A_3,1)\}$ corresponds uniquely to the student $\{(A_1, 0.25), (A_2, 0.25), (A_3, 0)\}$, whose profile is $s_i = (b, b, a)$. In other words, the A. Jones technique is equivalent with Voskoglou's method for the students' individual assessment. The only difference is that the former expresses the fuzzy data with numerical values, while the latter expresses it qualitatively in terms of the fuzzy linguistic labels of U.

Notice also that the teacher may put a target for his/her class and may establish didactic strategies in order to achieve it. For example he/she may ask for the deviation, say D, with respect to the teacher to be $0.25 \le D \le 0.5$, for all students and in all steps. In this case the application of the A. Jones technique could help the teacher to determine the divergences with respect to this target and hence to readapt his/her didactic plans in order to diminish these divergences.

Further details of the above fuzzy methods will be developed in the next section, when presenting the classroom experiment performed at the Faculty of Computer and Information Sciences of the Ain Shams University.

3. The Classroom Experiment

The subjects of the following experiment were 7 postgraduate students (M.Sc. program) of the Faculty of Computer and Information Sciences of the Ain Shams University, Cairo, Egypt. Obviously the above students had a rich background and experience on the use of computers in general and on applying computer techniques for modelling real world problems in particular. Also their mathematical background was strong enough, since they had attended four courses on Mathematics and two courses on Probability and Statistics during the first two years of their undergraduate studies, i.e. 2-3 years ago. This means of course that their knowledge on these subjects was not so fresh.

Prof. Salem, the second of the authors of the present paper, being the tutor of the course of "Advanced Artificial Intelligence" of the M. Sc. program, introduced to the subjects of our experiment the basic ideas of papers [1] and [4] and then he asked them to solve the 10 problems contained in the list of the Appendix of [1]. We recall that the mathematical topics related to these problems included elementary and linear algebra, differential and integral calculus, elementary differential equations and probability theory. The experiment was performed under the same conditions described in section 4 of [1]. The two authors marked together the students' papers and they applied Voskoglou's fuzzy methods in assessing their performance as follows:

Individual assessment

According to the marks obtained the student profiles were the following: $s_1 = (a, a, a)$, $s_2 = (b, b, a)$, $s_3 = (c, b, b)$, $s_4 = (c, c, b)$, $s_5 = (c, c, c)$, $s_6 = (d, c, c)$, $s_7 = (d, d, d)$. In this particular case the above profiles define a relationship of total order among the students concerning their performance given by: $s_1 < s_2 < s_3 < s_4 < s_5 < s_6 < s_7$.

The corresponding deviations with respect to the teacher are:

$$D_1 = \{(A_1, 1), (A_2, 1), (A_3, 1)\},\ D_2 = \{(A_1, 0.75), (A_2, 0.75), (A_3, 1)\},$$
$$D_3 = \{(A_1, 0.5), (A_2, 0.75), (A_3, 0.75)\},\ D_4 = \{(A_1, 0.5), (A_2, 0.5), (A_3, 0.75)\},$$
$$D_5 = \{(A_1, 0.5), (A_2, 0.5), (A_3, 0.5)\},\ D_6 = \{(A_1, 0.25), (A_2, 0.5), (A_3, 0.5)\},$$
$$D_7 = \{(A_1, 0.25), (A_2, 0.25), (A_3, 0.25)\}.$$

Group's assessment

Since the total number of the group's students is $n=7$, the membership function m_{A_i}, $i=1, 2, 3$, is defined by

$$m_{A_i}(x) = \begin{cases} 1, & if \ \frac{28}{5} < n_{ix} \le 7, \ i.e. \ n_{ix} = 6, \ 7 \\ 0,75, & if \ \frac{21}{5} < n_{ix} \le \frac{28}{5}, \ i.e. \ n_{ix} = 5 \\ 0,5, & if \ \frac{14}{5} < n_{ix} \le \frac{21}{5}, \ i.e. \ n_{ix} = 3, \ 4 \\ 0,25, & if \ \frac{7}{5} < n_{ix} \le \frac{14}{5}, \ i.e \ n_{ix} = 2 \\ 0, & if \ 0 \le n_{ix} \le \frac{7}{5}, \ i.e \ n_{ix} = 0, \ 1 \end{cases}$$

Observing the above students' profiles one finds that at the stage of analysis/mathematization *1, 1, 3* and *2* students demonstrated very low, low, intermediate and high performance respectively. The corresponding numbers for the stage of solution are *1, 2, 3, 1* and for the stage of validation/implementation are *2, 2, 2, 1*. Therefore the stages of the MM process are represented as fuzzy subsets of *U* in the form:

$A_1=\{(a,0), (b,0), (c, 0.5), (d, 0.25), (e,0)\}$, $A_2=\{(a,0), (b, 0.25), (c, 0.5), (d, 0), (e,0)\}$, $A_3=\{(a, 0.25), (b, 0.25), (c, 0.25), (d, 0), (e,0)\}$.

The application of the centroid defuzzification technique on the above fuzzy data gives:

Stage A_1: $x_c = \frac{1}{2}[\frac{5(0.5)+7(0.25)}{0.5+0.25}] \approx 2.833$, Stage A_2: $x_c = \frac{1}{2}[\frac{3(0.25)+5(0.5)}{0.5+0.25}] \approx 2.167$,

Stage A_3: $x_c = \frac{1}{2}[\frac{0.25+3(0.25)+5(0.25)}{0.25+0.25+0.25}] = 1.5$

Next we calculated the membership degrees of the students' profiles and their possibilities, which are presented in Table 1. The outcomes of Table 1 were calculated with accuracy up to the third decimal point. Also, since the maximal membership degree is *0.062*, the possibilities were calculated by the formula $r_s = \frac{m_s}{0.062}$.

Table 1: Membership degrees and possibilities of the students' profiles

A_1	A_2	A_3	m_s	r_s
c	b	a	0.031	0.5
c	b	b	0.031	0.5
c	c	a	0.062	1
c	c	b	0.062	1
c	c	c	0.062	1
d	b	a	0.016	0.258
d	b	b	0.016	0.258
d	c	a	0.031	0.5
d	c	b	0.031	0.5
d	c	c	0.031	0.5

Remark: The students' profiles obtained by the data of our experiment enabled us to represent the stages A_i , $i=1,2,3$ of the MM process as fuzzy subsets of U. However the converse is not true. In fact, given the A_i*'s* one cannot determine uniquely the corresponding student group. For example, it is straightforward to check that the student profiles *(b, a, a), (c, b, a), (c, c, b), (c, c, c), (d, b, b), (d, c, e)* and *(e, e, e)* determine the same A_i*'s*. One could also construct groups with *n* students, $n \neq 7$, corresponding to the same A_i*'s*. This explains why the profiles of Table 1 differ from those obtained by the data of our experiment.-

According to the outcomes of the last column of Table 1 the group's ordered possibility distribution is *r:* $r_1= r_2= r_3=1, r_4= r_5= r_6= r_7= r_8=0.5, r_9= r_{10}=0.258$. Therefore the group's *strife (discord),* which expresses conflicts among the sizes (cardinalities) of the various sets of alternatives, is $ST(r)= \frac{1}{\log 2}[\sum_{i=2}^{10}(r_i - r_{i+1})\log \frac{i}{\sum_{j=1}^{i} r_j}]$

$$\approx \frac{1}{0.301}(0.5\log\frac{3}{3} + 0.242\log\frac{8}{5.5} + 0.258\log\frac{10}{6.016}) \approx$$

$\approx$3.32[(0.242)(0.163)+(0.258)(0.221)] $\approx$0.319 (see section 2 of [2] and p.28 of [13]).

Also the group's *non-specificity (imprecision),* which indicates that some alternatives were left unspecified is $N(r)= \frac{1}{\log 2}[\sum_{i=2}^{10}(r_i - r_{i+1})\log i]$

$$\approx 3.32(0.5\log 3 + 0.242\log 8 + 0.258\log 10)$$

$\approx$3.32[(0.5)(0.477)+(0.242)(0.903) +0.258)] $\approx$2.36 (see section 2 of [2] and p.28 of [13]).

Thus the group's *total possibilistic uncertainty* is $T(r)=ST(r) + N(r) \approx 2.679$

4. Comparison with Voskoglou's Experiments

The same experiment had been performed earlier by Voskoglou with two groups of students of the Graduate Technological Educational Institute (T. E. I.) of Patras, Greece. The first of them (G_1) was a group of 35 students of the School of Technological Applications, i.e. future engineers, being at their second term of studies. Part (about the 1/3) of the lectures and the exercises of mathematical courses for the students of this School are performed in a computer laboratory, where the instructor presents the corresponding mathematical topics in a more "live" and attractive to students' way, while the students themselves, divided in small groups, use already existing mathematical software to solve the problems with the help of computers. The second group (G_2) consisted of 50 students of the School of Management and Economics being also at their second term of studies. The topics covered in the mathematics course of the first term for G_2 were almost the same with those for G_1. The only difference was that the lectures in the mathematical courses for the students of the School of Management and Economics are performed in the classical way on the board including a number of exercises and examples connecting mathematics with real world applications and problems (for more details about these experiments see section 4 of [1]) and section 3 of [2]).

The values of *T(r)* were found to be 2.653 for G_1 and 2.611 for G_2 [2]. On comparing these values with the corresponding value 2.679 for the group of postgraduate students of the Ain Shams University (G_3), one can see that no significant differences appear concerning the average performances of the three groups in the experiment. In fact, the maximum difference of these values is 0.068, which is too small with respect to the values themselves.

The application of the centroid defuzzification technique for the groups G_1 and G_2 (see section 4 of [1]) and G_3 (see above) gave the following values for x_c:

Stage A_1: 3.25 (G_1), 2.5 (G_2), 2.833 (G_3)

Stage A_2: 2.833 (G_1), 1.75 (G_2), 2.167 (G_3)

Stage A_3: The same value 1.5 for all groups

Thus, according to our criterion (see section 2), at the stages A_1 and A_2 the group G_1 demonstrates a better performance than G_3, which demonstrates a better performance than G_2. Also the three groups demonstrate identical performances at the stage A_3.

5. Discussion and Conclusions

In the present paper we compared the performances of the student groups' G_1, G_2 (T.E.I. of Patras, Greece) and G_3 (Ain Shams University, Egypt) in solving 10 problems involving MM by using the Voskoglou's fuzzy methods developed in [1] and [2]. The students of G_1 and G_2 had attended recently the mathematical course corresponding to the topics of these problems, in contrast to those of G_3, who attended such courses 2-3 years ago. Therefore it was normally expected for G_1 and G_2 to demonstrate a better performance than G_3.

Calculating the groups' total possibilistic uncertainty we found that no significant differences appeared with respect to the average performances of the three groups. On the contrary, using the centroid technique we found that the weighted average performance of G_1 (in which the results of the performance close to the ideal one have much more weight than those close to the lower end) at the first two stages of the MM process was better than that of G_3, which was better than the corresponding performance of G_2. We also found that the three groups demonstrated identical performances at the third and last stage of the MM process.

Combining the above findings to the fact that the corresponding mathematical course for G_1 was performed using the computers as a basic tool and that the students of G_3 had a rich experience in using the computer techniques, we reach to the conclusion that very possibly *the use of computers as a tool for problem solving enhances the students' abilities in solving real world problems involving mathematical modelling.* This is crossed by the experimental results presented in Voskoglou's ([1] and [4]) and in other researchers' [15-16] earlier works.

However, we believe that there is a need for further experimental studies on the subject, in order to obtain stronger and therefore safer statistical data. This is actually one of our priorities in our plans for further future research on MM.

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