Enhancement of Positron Emission Tomography (PET) Image Based on Thinning Algorithm and Shocking Filter

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Abstract

In this paper, we present an image de-blurring algorithm for Positron Emission Tomography (PET). PET medical image is fuzzy and blurred. The PET image needs to be enhancedin order to reduce the need for using another scanning machine like Computed Tomography and MRIthat areusually combined with it in the real life. This paper presents a novel approach to apply the Thinning Algorithm as well asa new hybrid approach to the combination of two different Enhancing Algorithms in order to enhance the PET image, Thinning algorithm, and Shocking Filter. Thinning or Skeleton Algorithm, based on the ideas of dilation and erosion, reduces the thickness of a given shape by finding its internal line without destroying its meaning. The function of the Shocking Filter or Coherence Filter is de-noising, de-blurring and sharpening the image. Thinning or Skeleton Algorithms were combined with a Shocking Filter or Coherence enhancing Filter to detail the image and define the inside structure of the body. The experimental results show the difference between the hybrid approaches by analyzing their de-blurring behaviors.

Keywords :*PET Image, Image De-blurring, Thinning Algorithm, Skeleton Algorithm, Shock Filter, Coherence enhance*

1. Introduction

Positron Emission Tomography (PET) [1, 2, 3] is a medical term referring to a specific type of medical image or medical equipment. The produced PET image is fuzzy and blurred. It does not give specific details about the structural detail of the body; it reveals tumors and malignant tumors as black spots. Improvements in imaging techniques and instrumentation have revolutionized early diagnosis and treatment of a number of different pathological conditions.

Thinning Algorithm methodology is widely used in the enhancement ofan image. It is illustrated in many studies of medical image production (CT, MRI, etc). It could be parallel [4], sequential [5, 6], or maximum method [7, 8, 9] and using different mask values (3x3, 6x6, 8x8). It is reliable, efficient and mainly used to produce the overall shape or structure of the image. Skeleton Algorithm is another methodology that is used to improve the quality of an image.Skeleton is the process of peeling off a pattern as many pixels as possible without affecting the general shape of the pattern [10].

On the other hand, Shocking isalso used to enhance images and improve quality. Kramer and Bruckner [11] proposed Shock Filter as a 1D filter based on the idea of dilation process near maximum and an erosion process near minimum. The decision made by Laplace if (-) is considered to be maximum, (+) consider to be minimum (Perform Sharpening). As time passed, interesting modifications of Shocking Filter were presented [12, 13, 14]. Many studies have shown the Shocking Filter with different equations and different characteristics. Some of these studies depend on noise removal [15] and others on de-blurring and sharpening [16], [17]. Coherence Enhancement Shock Filter [18] is another model of Shock Filter. It is used to enhance the coherent flow like structural by combining the Shock Filter with rebuts orientation estimation by means of structure tensor.

In this paper, we present an image de-blurring algorithm for Positron Emission Tomography (PET). We present a novel approach to apply the Thinning Algorithm or Skeleton Algorithm through a series of binary images, and besides, our studies develop a novel approach that combines Thinning Algorithm or Skeleton Algorithm with theShock Filter or Coherence Filter to sharpen or smooth the edges and solve blurring.

This paper is organized as follows. Section twodescribes the Thinning and Skeleton Algorithm. Section Three provides a brief summary or overview about Shocking Filter and Coherence Enhancing. Section four presents the proposed enhancement approach. Section five describesthe image quality measurement.Section six shows the experiments' results.Finally, Section nine presents our conclusions.

2. Morphological Image Processing

It is a difficult task to identify an object within an image. One way to simplify this problem is to change the grayscale image into a binary image, in which each pixel is identified by a value of either 0 or 1. Most morphological algorithms are asimple logic operation [10].Thinning and Skeleton are two kinds of Morphological Image Processing.

2.1 Thinning algorithms

Thinning Algorithm is a morphological operation that isused to remove selected foreground pixels from binary images and identify an object within an image. Thinning is somewhat like erosion. It is commonly used to tidy up the output of edge detectors by reducing all lines to asingle pixel thickness [6, 8]. Like other morphological operators, thinning operators take two pieces of data as input. One is the input image, which maybe either binary or grey scale. The other is the structuring element, which determines the precise details of the effect of the operator on the image [8]. It is divided into two classes: iterative and non-iterative. Non-iterative does not always produce accurate results,but it is faster than iterative [9]. Iterative and non-iterative have different performing methods with different rules and operations.

Almost all Iterative Thinning Algorithms depend on peeling the image until revealing the final structure of the image. Numbers of steps dothat, and it depends on the Connectivity Numbers. The Connectivity Numbers (CN)arerepresented by the following equation:

$$C_n = \sum_{i=0}^{n} P_n - (P_n * P_{n-1} * P_{n-2}) \qquad (1)$$

Where n represents the number of pixelsand P_n represents the color of eight neighbors of the pixel analyzed. 0 is the value of backgroundand 1 is representing the value of the image (foreground).

In this paper, we have applied the Zhang SuenIterative Thinning Algorithm [6]. That algorithm is iterative parallel made by two sub iterations. For the 1st iteration, a pixel n (i , j) is deleted if it satisfies the following condition:

1. Its connectivity number is one.
2. It has at least two black neighbors and not more than six.
3. At least one of n (i, j+1), n (i-1,j), and n (i, j-1) is white.
4. At least one of n (i-1, j), n (i+1, j), and n (i, j-1) is white.

In the second sub iteration, it depends on the first and second privies condition and on another two conditions:

1. At least one of n (i-1,j), n (i,j+1), and n (i+1,j) is white.
2. At least one of n (i,j+1), n (i+1,j), and n (i,j-1) is white.

When a pixel meets those conditions, it will be deleting. Algorithm stops when there are no more erasable pixels.

2.2 Skeleton algorithm

Thinned objects sometimes have the appearance of a skeleton, but they are not always uniquely defined [19]. Blum has introduced a skeletonizing technique called Medial Axis Transformation that produces a unique skeleton for a given object [19]. The Skeleton technique was first introduced by Blom (1967) [19]. Skeleton is the representation of an object that can have its internal structure.It is close in description to Thinning but the output of both of them is unique and different. In other words, the original shape could be reconstructed using skeleton points and their distance to shape boundary, and this couldn't be done in a thinned image after performing a Thinning Algorithm. The Skeleton technique depends on MedialAxis Transformation (MAT) that helps to produce a unique Skeleton for a given object. The Medial Axis Skeleton consists of the set of points that are equally distant from two closest points of an object boundary as shown in the Fig. (1). Grass Fire Analogy is a definition of Medial Axis Transformation. Each point of the Grass Fire boundary is considering a point fire, and all points are burning with the same intensity. The fire spreads in a circle from the boundary point at which it starts and burns with a constant rate of one unit distance per unit time, so that at time t the outer extent of the burned area is the curve parallel to the boundary but offset by distance t. The Medial Axis consists of the closure of the quench points of the fire. Quench Pointsare the points where fire beginsat two or more different boundary points that meet and cover one another. By the time the fires meet it has define the third coordinate of the MAT.

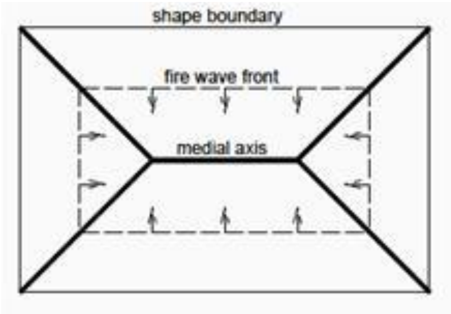


Fig.(1). Represents Medial Axis Transformation.

3. Shock Filter and Coherence Enhancing Filter

3.1 Shock filter

Shocking Filter is a type of Morphological Image Enhancement. Osher and Rudin introduced another idea of Shocking Filter for 1D and 2D [20]. They proposed a continuous class of filtering based on PDEs (Partial Differential Equations) and defined the *minmod* numerical scheme for avoiding any instabilities of the algorithm. Second order PDEs attempt to preserve the edge by evaluating the absolute value of the gradient of the image intensity. Performing PDEs is crucial for creating a sharp shock between two influence zones and producing piece-wise constant segmentation [15, 16]. In other words, the PDE's expressing the Shock Filter to use the zero crossing of the Laplacian as edge detector.

As for the 1D Shock Filters, they force the signal propagation speed to depend on the signal itself that done by sign of the second spatial derivative. Let's assume that u(x, y, t) represents an image. When t=0, u(x, y, 0) = f(x, y), represent the original image. When t >0 the equation for 1D will be

$$u_t = -\mathrm{sign}(u_{xx}) \qquad (2)$$

According to the first part of the eq. (2), the value of u_{xx} could be decompose to the following

$$u_{xx} = \begin{cases} -|u_{xx}| & \text{if } u_{xx} > 0 \\ |u_{xx}| & \text{if } u_{xx} < 0 \\ 0 & \text{if } u_{xx} = 0 \end{cases} \qquad (3)$$

The 2D shock filter of Osher and Rudin

$$u_t = -F\left(u_{\eta\eta}\right)|\nabla u_{xx}| \qquad (4)$$

Assuming F (0,0)=(0,0) and F(x,y) × (x,y) ≥ 0 ,and F(x,y) =(sign(x), sign (y)). Again with initial condition u(x, y, t=0) = u_0, and the gradient direction $\eta = \nabla u/\|\nabla u\|$. The function of Laplace used in the first part of the equation is a second order, (generally) nonlinear elliptic operator. So when Laplace goes to zero then the edge formation and sharpening process will occur, so the choice of Laplacian is rolled by how the zero-crossing of the differential operator defines edge of the processed image. That led to add 2D Gaussian to Laplace part. The equation will be as the following

$$\mathcal{L}(u) = \nabla^2 \left(u(x,y) * G(x,y)\right) \qquad (5)$$

The zero crossing depends on the width of the Gaussian Support [-σ, +σ]. The influence point of the entity (Gaussian part) can change according to σ, which makes the identification of the influence point less accurate [15].

3.2 Coherence enhancingshock filter

CoherenceenhancingFfiltering is a specific technique within the general classification of diffusionfiltering. diffusionfiltering, which models the diffusion process, is an iterative approach of spatialfiltering in which image intensities in a local neighborhood are utilized to compute new intensity values. Coherence enhancingfiltering is a regularized *nonlinear* diffusion that attempts to smooth the image in the direction using similar intensity values. *Nonlinear* diffusion means that the filter coefficients change in response to differential structures within the image. Coherence enhancementfilters take the novel Shock filter and apply a structure tensor equation [18]

$$u_t = -\text{sign}(v_{ww})|\nabla u_{xx}| \qquad (6)$$

where

$$v = G_\sigma * u \qquad (7)$$

$$w = J\rho(\nabla u) \qquad (8)$$

and the structure tensor formula

$$J\rho(\nabla u) = G_\rho * (\nabla\, u\, \nabla u^T) \qquad (9)$$

where w is the normalized dominant eigenvector of the structure tensor as stated in eq. (9).structuretensor is matrix 2x2. It was guaranteed to create a shock orthogonal to the flow direction of the pattern called Shock direction and maximizes the contrast differences. The flow direction, dilation, and erosion take place after the time structure becomes constant along the different flow directions then the sharp shock takes form orthogonal to it. By time t evolution reaches a piece-wise constant segmentation where coherentenhancement takes place. The structuretensorscale σ determines the size of the resulting flow patterns. The integration scale ρ helps to stabilize the directional behavior of the filter. For a better result, the integration scale must be greater than the structure scale.

4. The Proposed Enhancement Approach

Thinning algorithm apply to binary image. Many details exist in the PET image due to the intensity of body texture. Therefore, we present a novel approach to apply the Thinning algorithm through a series of binary images. After that the output of the Thinning or skeletonalgorithmsis considered as the input to the Shock filter or coherencefilter to produce a hybrid algorithm. Fig. (2) shows a diagram of the proposedapproach. The proposed approach is:

1. Use a series of threshold values to convert the original PET image to many binary images. For each image, we apply connectivity function, and then we combine all the produced binary images in one image.
2. Perform the Thinning algorithm on the final binary PET images.

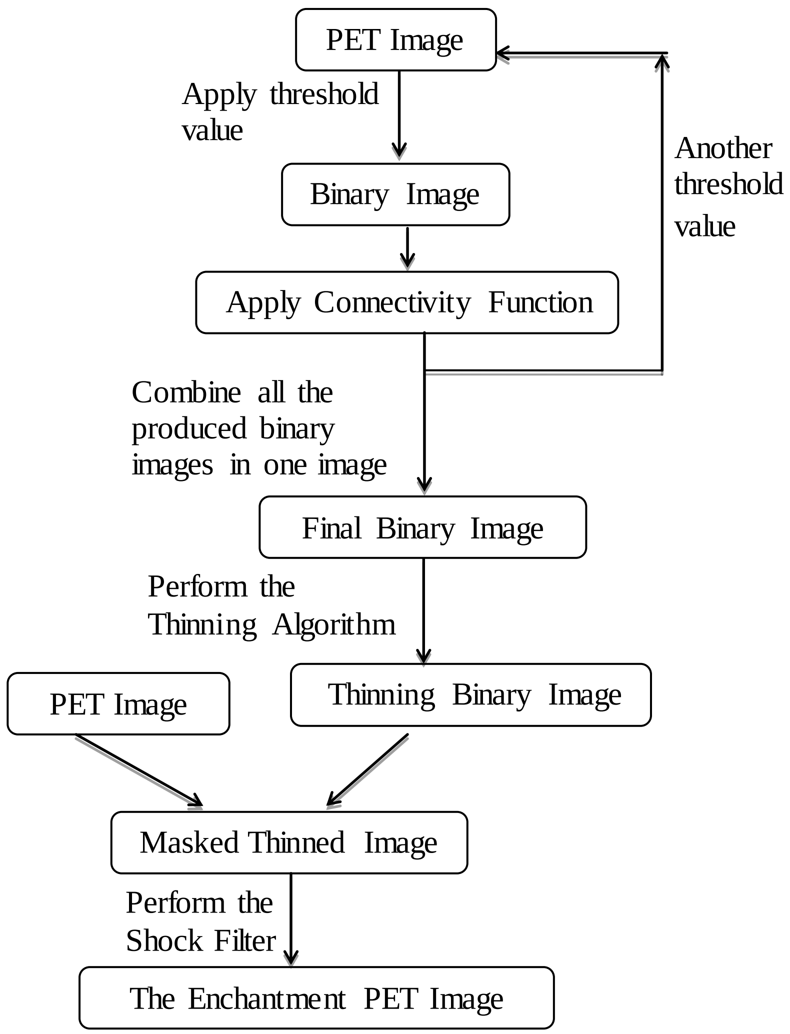


Fig.(2). TheProposed Enhancement Approach.

3. Combine the produced binary image and the original PET image, compare the corresponding pixels in the two images; if pixel value is 1 in the binary image, then the corresponding pixel in the PET image will set to 1. This overcomes the difference in data representation in the two images and allows applying the Shock filter after the Thinning algorithm.
4. Perform the Shock filter to the Masked thinned Image.

Repeat the previous proposed approach three times using the alternative Skeleton algorithm instead of the Thinning algorithm, and the alternative coherence filter instead of the Shock filter.

5. Image Quality Measurement

There are different ways to represent the quality of the image based on pixel distance, correlation, degree of similarity, and mean square error [8]. Let I denote the reference image and K denote the tested image.

Peak Signal to Noise Ratio (PSNR: It is used to measure the quality of image reconstruction after compression. The high value of PSNR means goodimage resolution.

$$\mathrm{PSNR} = 10 \log_{10} \frac{(l-1)^2}{\mathrm{MSN}} \qquad (10)$$

Where L represents the maximum number of pixel and the MSN represents themean square error with equation.

$$\mathrm{MEN} = \frac{1}{mn} \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} [I(i,j) - K(i,j)]^2 \qquad (11)$$

Quality of Image: The quality of image can be measured bydefining the cosine angle between the tested image and the reference one. The cosine angle defines the degree of similarity between the tested image and the reference one.

$$\cos(I,K) = \frac{I'^{T}K}{\|I\|\|K\|} \qquad (12)$$

Cosine 1 indicates that the two images are the same,meanwhile0 shows that the two images are different.

Root Mean Square Deviation (*RMSD*) also called *L2D* (*L2 Euclidean distance*)meansthe measurement of the difference between two vectors.

$$\mathrm{RMSD} = \sqrt{\mathrm{MEN}} \qquad (13)$$

Where MEN as it was defined in eq. (11).

6. Experiments and Results

First, we apply our approach on PET image. Second, the performance of the proposed approach is measured by applying it on a fingerprint image.

6. 1 Results with PETimage

First, we apply the proposed approach on PET image of size 441x264 pixels produced from Philips Scanner Equipment. Fig. (3) shows the PET image. The proposed approach will be used to define the structure of PET image by sharpening the edges of the internal structure of the human body. Different values of threshold were used to convert the original PET image to binary images. The threshold values starting from 0.1 up to 0.9 as shown in Fig. (4). One can see from Fig. (4) that each threshold represents a different structural part of human body due to intensity of body texture. After that, we have determined the perimeter pixels of the objects in the binary images in Fig. (4). Fig. (5) shows the edge images representation. Each binary image defines a different structural part of the human body.

Fig.(3). Original PET Image.

Fig. 6 (a) shows the final binary image after combining all the binary images in Fig. (5). Fig 6(b) and 6(c) show the binary images after performing the Thinning algorithm and the Skeleton algorithm on the final binary image, respectively. After that, we have combined the original PET image with the binary Skeleton or with the binary Thinned image in order to enhance the structural edges of the body as the first step of sharpening the edges. That was done by comparing the corresponding pixels in the two images; if pixel value is 1 in the binary image, then the corresponding pixel in the PET image will set to 1, as shown in Fig. 7(a) and 7(b) respectively. This overcomes the difference in data representation in the two images and allows applying the Shock filter after the Skeleton or Thinning algorithm.

The images are now ready to be filtered using Shock filter. The next experiments were performed to compare between applying the Shock filter to the original PET image and to the masked PET image. Many parameters control the quality of image that is produced from Shock filter, number of iterations, Gaussian matrix, and time steps. The first experiment performs the Shock filter on the original PET image using different number of iteration, 10, 20, 30, 40, and 50, Gaussian matrix equals to 7 and time steps equals to 0.25. Fig. (8) shows the results of Shock filter that were applied to the original PET image through a series of different numbers of iterations. One can see from Fig. 8 that the internal body structural edge get sharpened by increasing the number of iteration.

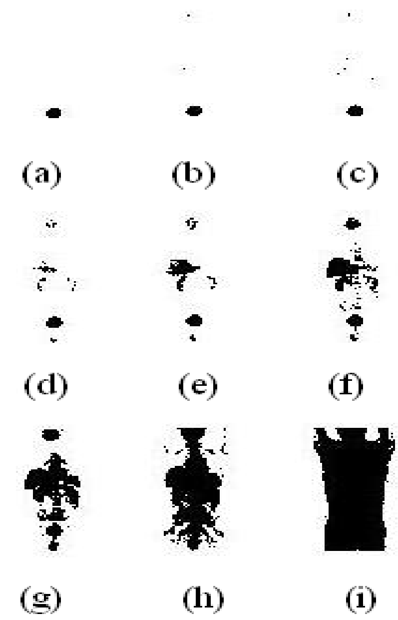


Fig. (4). The images from left to right and from top to down show the binary images that was produced by using different threshold values starting from 0.1 up to 0.9.

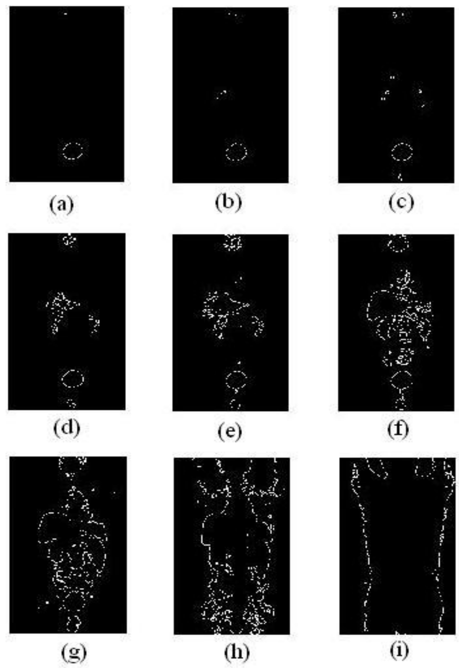
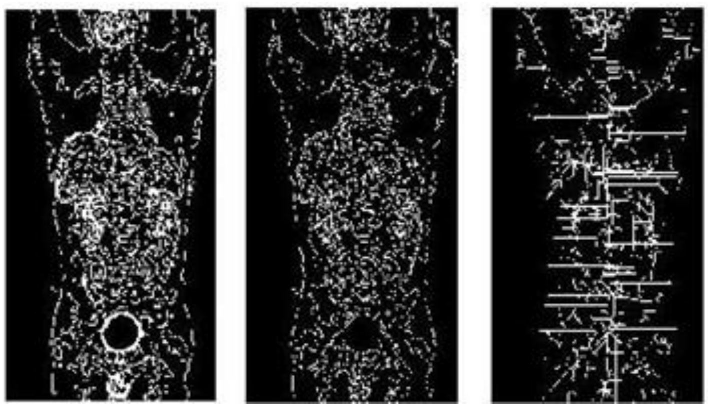
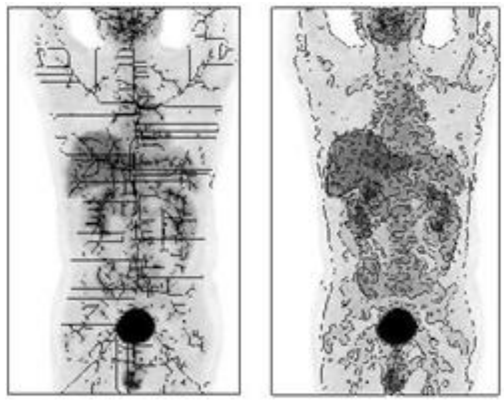


Fig. (5). A series of images show the edge detection for each threshold image in Fig. (4).



(a) (b) (c)

Fig. (6). (a) The final binary image, and The binary image after applying the (b) Thinning Algorithm and (c) Skeleton Algorithm.



(a) (b)

Fig. (7). (a) Masked original PET image with Skeleton image, (b) Masked original PET image with Thinned image.

The second experiment performs the Shock filter on the masked PET image. The parameters of Shock filter have the same values as pervious experiment. Fig. (9) shows the performance of applying the Shock filter on the Skeleton masked PET image. Fig. (10) shows the performance of applying the Shock filter on the thinned masked PET image.

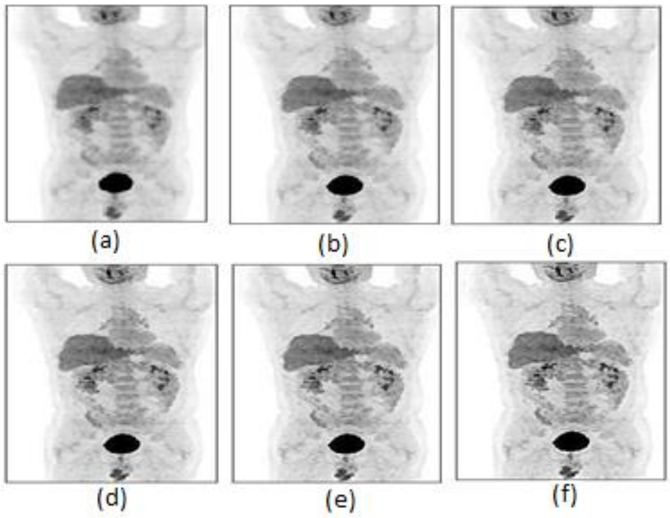


Fig.(8). De-blurring of original PET image. (a) Original image; (b) to (f) results of applying the Shock Filter through a number of iterations starting from 10 to 50.

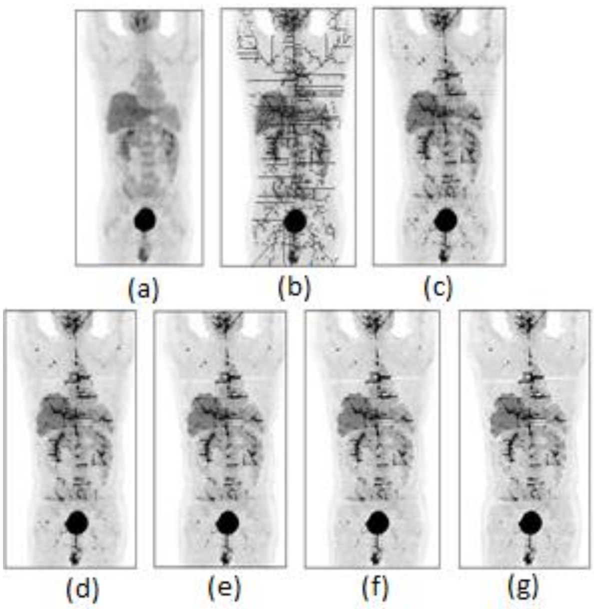


Fig. (9). De-blurring of Skeleton Masked PET image. (a) Original image; (b) Skeleton Masked PET image; (c) to (g) results of applying the Shock Filter through a number of iterations starting from 10 to 50.

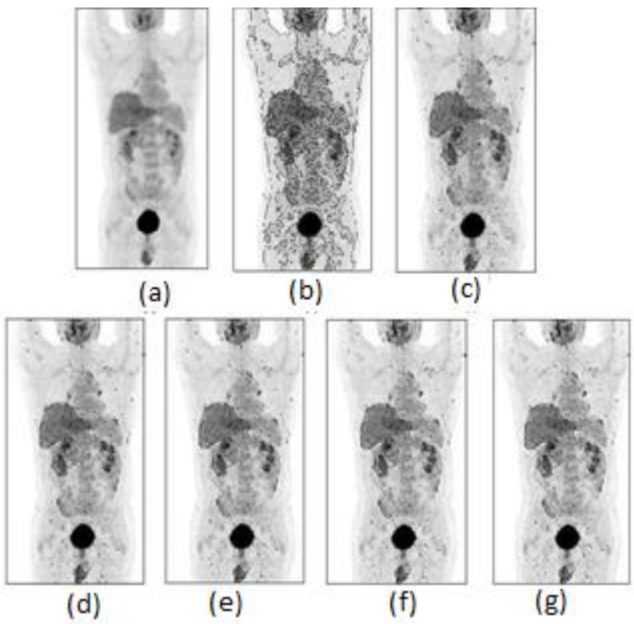


Fig. (10). De-blurring of Thinned masked PET image. (a) Original image; (b) Thinned Masked PET image; (c) to (h) results of applying the Shock Filter through a number of iterations starting from 10 to 50.

One can see from Fig. 9 and 10 that the lines that represent the skeleton or the thinning,have fused with the main image and lift some black dots, and that was undesired result. In addition, by increasing numbers of iteration, some of those dots vanish or lighten. The final experiment applies the Coherence enhancement filter on the Thinned masked and Skeleton masked PET image. Again, many parameters control the quality of image produced from Coherence enhancement filter as same as Shock filter and the values of eigenvalues and eigenvectors.Many parameters control the Gaussian derivative, degree of derivative, pattern emerging, and polynomial order. Fig. 11(a) shows the original PET image. The first and second Gaussian derivative at σ equals 3 are shown in Fig. (11). Fig. 11(b) and 11(c) show the first Gaussian derivative with respect to x and y at first polynomial order respectively. The second derivative with respect to x^2, y^2, and respect to xy is illustrated in Fig. 11(d), 11(e), and 11(f) respectively. Fig. (11) shows that by increasing the polynomial order of the Gaussian, the details of image is lost. So using second Gaussian derivative helps smooth the internal structure ofthe body without destroying it.

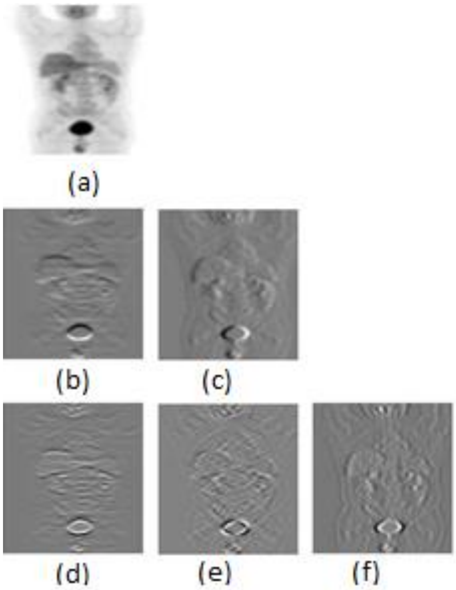


Fig. (11). (a) The original PET image. (b) and (c) the 1st Gaussian derivative. (d), (e) and (f) the 2nd Gaussian derivative.

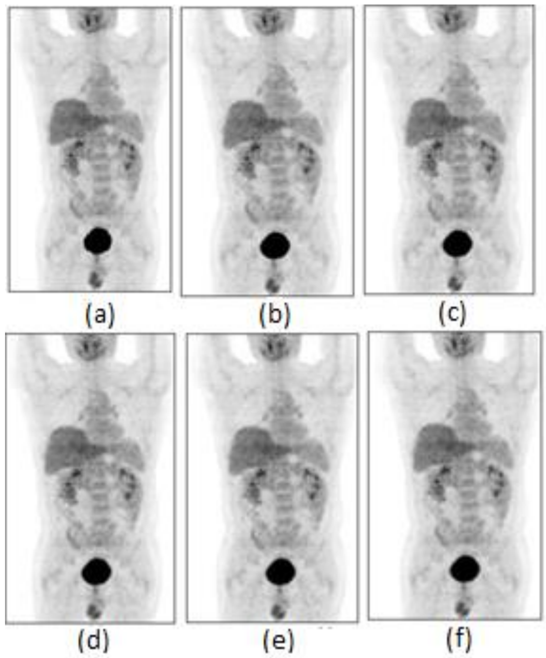


Fig. (12). De-blurring of original PET image. (a) Original image. (b) to (f) results of applying the Coherence Filter through a number of iterations starting from 10 to 50.

The Original PET image was tested by Coherence Filter as shown in Fig. (12).Fig. (13) represents the Skeleton Masked PET image after applying the Coherence Filter using different numbers of iterations. Fig. (14) shows the Thinned Masked PET image filtered by Coherent Enhancing in different number of iteration. Starting from Fig. (13)to(14), we can see that the local orientation is totally the same; the lines that represent the Skeleton and the Thinned Algorithm face slight changes in their pattern. One can see in Fig. (12), (13), and (14) that the increased number of iterations do not affect the performance of Coherence Filter. One can observe from Fig. (8), (9), (10), (12), (13), and (14) that the Shock Filter sharps the edges of the body structure andcombines the lines produced of the Thinned or Skeleton Algorithm. On the other side, the Coherence Enhancing smoothes the lines produced by the algorithms and has slight change on the structural content of the body.

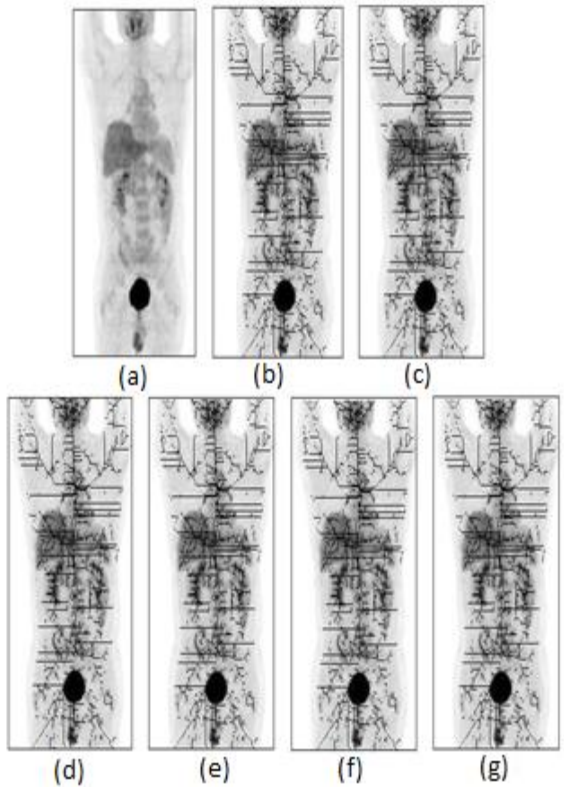


Fig. (13). De-blurring of Skeleton Masked PET image. (a) Original image; (b) Skeleton Masked PET image; (c) to (g) results of applying the Coherence Filter through a number of iterations starting from 10 to 50.

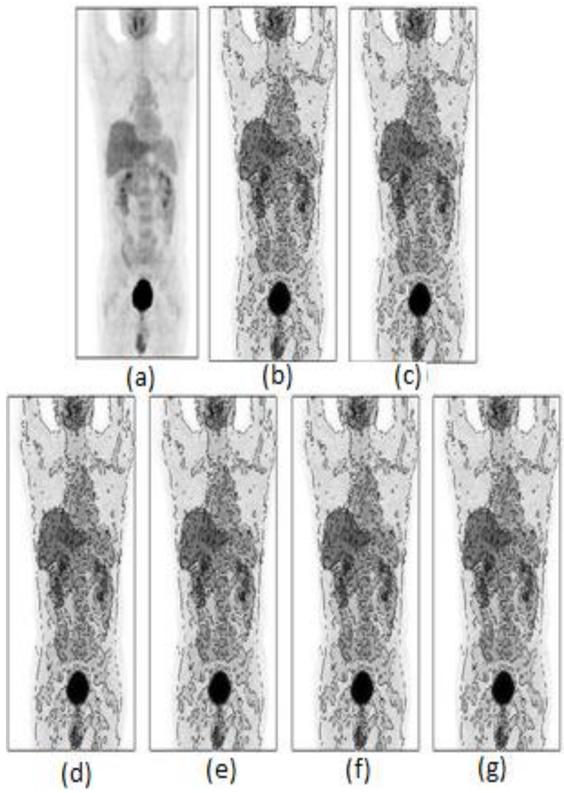


Fig. (14). De-blurring of Thinned Masked PET image. (a) Original image; (b) Thinned Masked PET image; (c) to (g) results of applying the Coherence Filter through a number of iterations starting from 10 to 50.

6.2 Results with Fingerprint Image

In this experiment, we have compared performances of the proposed approaches to the blurred fingerprint image, which are shown in Fig.15(b). The blurred fingerprint image is obtained from white Gaussian noise blur with SNR= 5dB and with the power density = 2dbw. The same experimentwas applied to the fingerprint image as it was applied to the PET image. First, we have extractedbinary image by using different values of thresholds as illustrated in Fig. (16). Fig. (17) shows the edge images representation for every threshold binary images represented in Fig. (16). Each binary image is defined in a different structure.

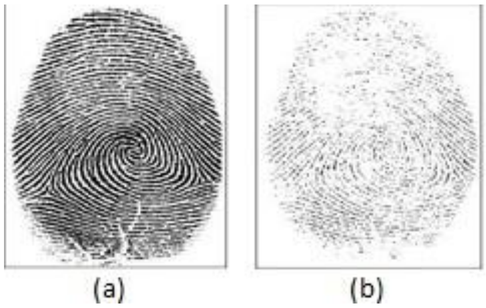


Fig. (15). (a) represents the original fingerprint image; (b) shows the fingerprint image after applying 5db White Gaussian noise.

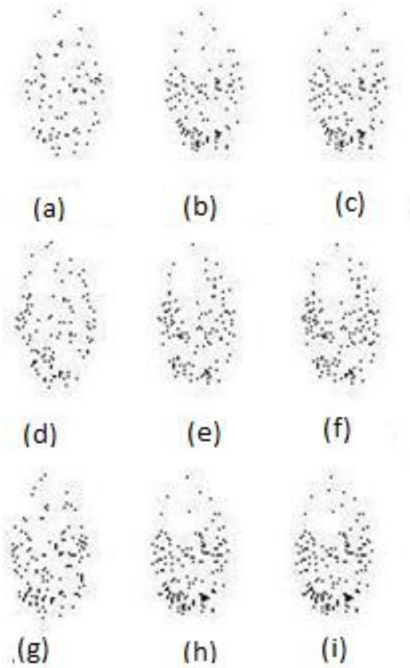


Fig. (16). The images from left to right and from top to down show the images that produced from using different threshold values started from 0.1 up to 0.9.

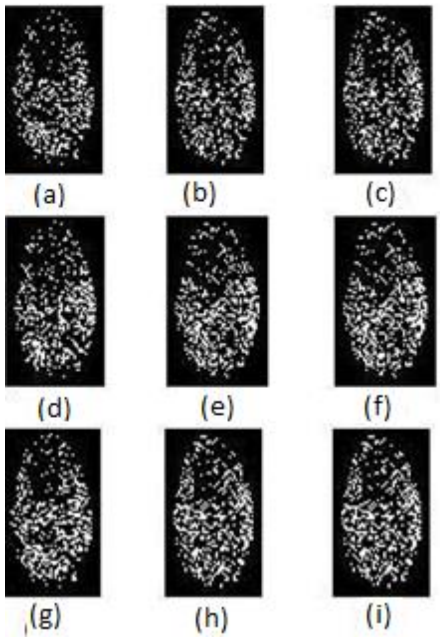


Fig. (167). A series of images to show the edge detection for each Threshold shown in Fig. (16).

Then, we combined all the binary images in Fig. (17) into final binary image as show in Fig. 18(a). Fig.18(b) and 18(c) show the binary images after performing the Skeleton Algorithm and the Thinning Algorithm on the final binary image, respectively. After that, we combined the original blurred fingerprint image with the binary Skeleton or with the binary Thinned image as shown in the Fig.19(a) and 19(b) respectively. Finally, we applied the Shock Filter and Coherence Filter using the Masked Fingerprint Image. The results are illustrated from a close view in Fig. (20) and (21), respectively. To study the effect of using Thinning Algorithm or Skeleton Algorithm before applying the Shock Filter or Coherence Filter, we have applied the Shock Filter or Coherence Filter to the original blurred fingerprint image. Fig. (22) shows the blurred image after applying shock Filter and Coherence Filter, respectively.

(a) (b) (c)

Fig. 17 (a) The Final Binary Image, and The Binary Image after applying the (b) Thinning Algorithm and (c) Skeleton Algorithm.

(a) (b)

Fig. 18. (a) Masked the Blurred Fingerprint Image with the Skeleton Image in Fig. 18(c), (b) Masked the Blurred Fingerprint Image with the Thinned image in Fig. 18(b).

(a) (b)

Fig. (19). De-blurring using Shock Filter with 30 iteration after applying on (a) Skeleton Masked Fingerprint Image. (b) Thinning Masked Fingerprint Image.

(a) (b)

Fig. (20). De-blurring using Coherence Filter with 30 iteration after applying on (a) Skeleton Masked Fingerprint Image. (b) Thinning Masked Fingerprint Image.

(a) (b)

Fig.(21). De-bulrred the Original Image using (a) Shock Filter. (b) Coherence Filter.

Finally, Table 1 compares between the proposed approaches using the PSNR, quality of image, and RMSD. PSNR and quality of image show that de-blurring the fingerprint image using the Skeleton/coherence or Thinning /coherence is better than Skeleton/Shock or Thinning/Shock (high PSNR means high resolution or quality). The coherence seems to have the advantage of de-blurring the image. Also the RMSD shows that the Coherence filter have lower results than Shock filter and this indicates that the feature retrieved of blurred image using coherence filter is more than Shock filter (high RMSD means low image enhancing).

The result of coherence and Shock filter state that coherence is much better for image enhancement than Shock filter.

Table (1).Performance criteria for de-blurring the blurred fingerprint image.

	Tested Algorithms					
	shock filter	***coherence filter***	***Skeleton/ Shock***	***Skeleton/ coherence***	***Thinning /Shock***	***Thinning / coherence***
PSNR (dB)	14.5095	23.9103	14.3948	15.0592	10.7213	12.0922
Quality of Image	0.0067	0.0077	0.0073	0.0078	0.0074	0.0077
RMSD	47.9814	16.2564	48.6182	45.0382	74.2119	63.3767

7. Conclusions

In this paper, we have proposed a new approach to smooth and sharpen the edges of the PET images. We have combined thinningalgorithm or skeletonalgorithm with the Shock filter or the coherenceenhancing Shock filter. Our target is to sharpen and smooth the structural edges of the internal body of PET image. Combining Thinning or Skeleton followed by the coherencefilterhave shown quite improvement in sharpening and de-blurring than combining Thinning or Skeleton followed by the Shock filter due to the property of the structure tensor, that smoothes the edges without affecting the flow like structure of the segmentation of the image. A suggestion for improvements in future work, may be to use a different approach of Thinning or Skeleton algorithm [7, 21], or other filter generations [13].

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