Generalized (α,β) Entropy Based Edge Detection in Grayscale Images

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Abstract

The two-parametric (α,β) entropy is called Sharma-Mittal entropy. It generalized Boltzmann-Gibbs, Tsallis and Rènyi entropies since its (α,β) parameters generate these entropies in limiting cases. These entropies can be easily estimated using a kernel estimate. This makes their use by several researchers in image processing greatly appealing. In this paper, a novel entropic algorithm for images corrupted with noise based generalized (α,β) entropy is proposed. The entropic algorithm finds the edges by eliminating the noise from the image in order that the proper edges are determined. The proposed Entropic method is tested under noisy conditions on various images and also compared with standard edge detectors such as Prewitt, Sobel and Canny. Experimental results show that the proposed technique have a better performance and efficient to be used for the edge detection in images corrupted by Salt-and-Pepper noise than different progressive edge detector techniques

Keywords: *generalized* (α,β) *Entropy; Edge Detection; noisy Images*; *Threshold Value*

1. Introduction

Edge detection has been used vastly in many applications of image and signal processing [1]. Its use contains image segmentation, pattern recognition and scene analysis. The edges are also use to locate the objects in an image and measure their geometrical features. Hence, the detection of edges is an important identification and classification tool in computer vision. This topic has attracted several researchers and several achievements have been produced to explore new and more robust techniques [2, 3].

Natural images are apt to artifacts and noise. Salt & pepper noise is a style of noise normally seen on images. It is typically appeared as randomly occurring white and black pixels. Salt & pepper noise lurks into images in situations where quick transients, such as decoding error or faulty switching [4].

Many studies have been published in the field of image edge detection, which attests to its importance within the field of image processing. A large number edge detection algorithms have been proposed. Up to date, we can't say one of them is the "best" edge detector. A perfect edge detector should be fit to detect the edge for any style of image and ought show higher resistance to noise[5, 6, 7].

Examples of techniques to edge detection include algorithms such as the Prewitt and Sobel edge detectors [1] that are based on the first order derivative of the pixel intensities. The Laplacian-of-Gaussian (LoG) edge detector [8, 9] is another common approach, using instead the second order differential operators to detect the location of edges [10, 11].

However, all of these algorithms can to be sensitive to noise, which is high frequency phenomenon. To solve this problem Canny proposed an edge detector, which merges a smoothing function with zero crossing based edge detection [12]. Although it is more flexible to noise than the above mentioned algorithms, its performance is still not helpful when the noise level is high. There are many cases where sharp changes in color intensity do not correspond to object boundaries like recording noise, surface marking and uneven lighting conditions [2]

In 1948, Shannon introduced his major work[13, 14], since this time the entropy appears as an interesting tool in many areas of data processing. In 1975, Sharma-Mittal [15, 16] introduced a wider class of entropies known as (α,β)-entropies. The functionalities of (α,β)-entropies share the main properties of Shannon's entropy. Furthermore, the (α,β)-entropies can be easily evaluated using a kernel estimate. This makes their use appealing in many areas of image processing [17, 18]. In this paper, we present an efficient entropic approach for edge detection images which utilizes generalized Sharma-Mittal entropy. Our technique for images edge detection has a relatively good performance in comparison to other techniques.

The organization of this paper is as follows. The next section discusses the generalized form of (α,β)-entropies especially generalized Sharma-Mittal entropy. The proposed entropic method is explained in Section 3. In Section 4, the details of the edge detection algorithm is described. Section 5 is to present the experimental results that validate the use of the proposed method. Advantages of our method and concluding remarks are outlined in Section 6.

2. Generalized Entropy

Physically Entropy can be associated with the amount of disorder in a physical system. In[13] Shannon redefined the entropy concept of Boltzmann/Gibbs as a measure of uncertainty regarding the information content of a system. Shannon defined an expression for measuring quantitatively the amount of information produced by a process. Then Entropy has applied across physics, mathematics, information theory and other branches of science and engineering [19]. When given a system whose exact description is not properly known, the entropy is defined as the expected amount of information needed to exactly specify the state of the system, given what we realize about the system.

Let $P = \{p_1, p_2, \cdots \cdots, p_k\}$ be the probability distribution of a discrete source. Therefore, $0 \le p_i \le 1, i = 1,2, \cdots, k$ and $\sum_{i=1}^{k} p_i = 1$, where k is the total number of states and P is called the entropy of the distribution. The entropy of a discrete source is often obtained from the probability distribution.

The Shannon Entropy can be defined as [13]

$$H(P) = -\sum_{i=1}^{k} p_i \ln(p_i) \quad (1)$$

This formalism has been shown to be restricted to the domain of validity of the Boltzmann–Gibbs–Shannon (BGS) statistics. Generally, systems that obey BGS statistics are called extensive systems. If we deem that a physical system can be decomposed into two statistical independent subsystems O and B, the probability of the composite system is $P^{O+B} = P^O \cdot P^B$, it has been verified that the Shannon entropy has the extensive property (additive):

$$H(O + B) = H(O) + H(B) \quad (2)$$

However, for non-extensive systems, some type of extension appears to become necessary. Sharma-Mittal entropy, which is useful for describing the non- extensive systems, is defined as Entropic edge detection for noisy images.

The generalized entropies of Sharma-Mittal of order α and type β is given by[15, 16]

$$H_{\alpha,\beta}(p) = \frac{1}{1-\alpha}\left[\left(\sum_{i=1}^{k} p_i^{\beta}\right)^{\left(\frac{1-\alpha}{1-\beta}\right)} - 1\right],\ \alpha \neq \beta, \alpha, \beta > 0 \tag{3}$$

In the limiting case, when α ⟶ 1, Sharma-Mittal entropy becomes Rènyi entropy [20, 21] which is

$$H_{\beta}^{R}(p) = \frac{1}{1-\beta} \ln \sum_{i=1}^{k} (p_i)^{\beta},\ \ \beta > 0\ ,$$

While for $\alpha \longrightarrow \beta$, it is Tsallis entropy [18, 22, 23] given by

$$H_{\beta}^{T}(p) = \frac{1 - \sum_{i=1}^{k} (p_i)^{\beta}}{\beta - 1}\ ,\ \beta > 0$$

In the limiting case when the two parameters (α,β) approach 1, we recover the Shannon entropy, as defined in (1).

Note that Sharma-Mittal entropy has a non-extensive property for two statistically independent systems, satisfied the following pseudo additivity entropic formula

$$H_{\alpha,\beta}(O+B) = H_{\alpha,\beta}(O) + H_{\alpha,\beta}(B) + (1-\alpha)\cdot H_{\alpha,\beta}(O)\cdot H_{\alpha,\beta}(B). \qquad (4).$$

3. Thresholding Value based Sharma-Mittal entropy

A gray level image can be clarified by an intensity function, which defines the gray level value for each pixel in the image. Particularly, in a digital image of size $M \times N$ an intensity function $f(x,y)$ $\{\, f(x,y) \mid x \in \{1,2,\cdots,M\},\ y \in \{1,2,\cdots,N\}\}$, takes as input a particular pixel from the image, and output its gray level value, which is usually in the range of 0 to 255 (if 256 levels are used) [24, 25].

Thresholding produces a new image based on the original one represented by f [26, 27]. It is basically another function $g(x,y)$, which produces a new image (i.e. *the thresholded image).* A threshold is calculated for each pixel value. This threshold is compared with the original image (i.e. f) to determine the new value of the current pixel. g can be represented by the following equation [28, 29].

$$g(x,y) = \begin{cases} 0, & \text{if } f(x,y) \le t \\ 1, & \text{if } f(x,y) > t \end{cases}\ ,t \text{ is the thresholding value.}$$

In image processing techniques, entropy measures the normality (i.e. normal or abnormal) of a particular gray level distribution of an image. While a whole image is considered, the Sharma-Mittal entropy as defined in (3) will indicate to what extent the intensity distribution is normal. When we extend this concept to image segmentation, i.e. dealing with Object and Background regions in an image, the entropy is determined for both regions, while the subsequent entropy value provides an indication of the normality of the segmentation. In that case, two equations are required for each region, every of them called priori.

When applying maximum entropy, in image thresholding, every gray level value is a candidate to be the value of thresholding. Each value will be applied to classify the pixels into two groups based on their gray levels and their affinity, as greater or less than the threshold value (t).

Let $p_1, p_2, \ldots, p_t, p_{t+1}, \ldots, p_k$ be the probability distribution for an image with k gray-levels, where p_t is the normalized histogram i.e. $p_t = h_t/(M \times N)$ and h_t is the gray level histogram [30, 31]. From this distribution, we may derive two probability distributions, one for the Object (class O) and the other for the Background (class B), are shown as follows:

$$p_O : \frac{p_1}{P_O}, \frac{p_2}{P_O}, \ldots, \frac{p_t}{P_O}, \quad P_O = \sum_{i=1}^{t} p_i$$

$$p_B : \frac{p_{t+1}}{P_B}, \frac{p_{t+2}}{P_B}, \ldots, \frac{p_k}{P_B}, \quad P_B = \sum_{i=t+1}^{k} p_i \text{ , } t \text{ is the threshold value.} \tag{5}$$

In terms of the definition of Sharma-Mittal entropy of order α and type β, the entropy of Object pixels and the entropy of Background pixels can be defined as follows:

$$H^{O}_{\alpha,\beta}(t) = \frac{1}{1-\alpha}\left[\left(\sum_{i=1}^{t}\left(\frac{p_i}{P_O}\right)^{\beta}\right)^{\left(\frac{1-\alpha}{1-\beta}\right)} - 1\right], \ \alpha \neq \beta, \alpha, \beta > 0$$

$$H^{B}_{\alpha,\beta}(t) = \frac{1}{1-\alpha}\left[\left(\sum_{i=t+1}^{k}\left(\frac{p_i}{P_B}\right)^{\beta}\right)^{\left(\frac{1-\alpha}{1-\beta}\right)} - 1\right], \ \alpha \neq \beta, \alpha, \beta > 0 \tag{6}$$

The Sharma-Mittal entropy $H_{\alpha,\beta}(t)$ is parametrically dependent upon the threshold value t for the object and background. It is formulated as the sum each entropy, so allowing the pseudo-additive property for statistically independent systems, as defined in (4). We try to maximize the information measure between the two classes (object and background). When $H_{\alpha,\beta}(t)$ is maximized, the brightness level t that maximizes the function is considered to be the optimum threshold value. This can be done with a cheap computational effort [24].

$$t^{opt} = \text{Arg}\max\left[H^{O}_{\alpha,\beta}(t) + H^{B}_{\alpha,\beta}(t) + (1-\alpha)\cdot H^{O}_{\alpha,\beta}(t)\cdot H^{B}_{\alpha,\beta}(t)\right]. \tag{7}$$

When $\alpha \longrightarrow 1$, the threshold value in (4), equals to the same value found by Shannon Entropy. Thus this suggested method includes Shannon's method as a special case. The following term can be used as a criterion function to obtain the optimal threshold at $\alpha \longrightarrow 1$.

$$t^{opt}_{Sh} = \text{Arg}\max\left[H^{O}_{\alpha,\beta}(t) + H^{B}_{\alpha,\beta}(t)\right]. \tag{8}$$

The Sharma-Mittal entropic Threshold algorithm to determine a suitable threshold value t^{opt} and α and β can be described as follows:

Algorithm 1: Sharma-Mittal Threshold Value Selection

1. Input: A digital grayscale noisy image I of size $M \times N$.
2. Let $f(x,y)$ be the original gray value of the pixel at the point $(x,y),(x=1,2,\cdots,M,\ y=1,2,\cdots,N)$
3. Calculate the probability distribution $p_i,\ \ 0 \le i \le 255$
4. For all $t \in \{0,1,\dots,255\}$,
  I. Apply Equation (5) to calculate $P_O,\ P_B, p_O$ and p_B
  II. ***if*** $0 < \alpha < 1$ ***then***
    Apply Equation (7) to calculate optimum threshold value t^{opt}.
  else
    Apply Equation (8) to calculate optimum threshold value t^{opt}_{Sh}.
  end-if
5. **Output**: The suitable threshold value t^{opt} of I, for $\alpha, \beta > 0,\ \alpha \ne \beta$

4. Edge Detection Algorithm

The process of spatial filtering consists simply of moving a filter mask w of order $m \times n$ from point to point in an image. At each point (x,y), the response of the filter at that point is calculated a predefined relationship. We will use the usual masks for detection the edges. Assume that $m = 2a + 1$ and $n = 2b + 1$, where a, b are nonnegative integers. For this purpose, smallest significant size of the mask is 3×3, as shown in Figure 1[1, 2, 32].

$w(-1,-1)$	$w(-1,0)$	$w(-1,1)$
$w(0,-1)$	$w(0,0)$	$w(0,1)$
$w(1,-1)$	$w(1,0)$	$w(1,1)$

Figure 1: Mask coefficients showing coordinate arrangement

$f(x-1,y-1)$	$f(x-1,y)$	$f(x-1,y+1)$
$f(x,y-1)$	$f\ (x,y)$	$f(x,y+1)$
$f(x+1,y-1)$	$f(x+1,y)$	$f(x+1,y+1)$

Figure 2: Image region under the 3×3 mask

Image region under the above mask is shown in Figure 2. In order that edge detection, at first classification of all pixels that satisfy the criterion of homogeneousness, and detection of every pixels on the borders between different homogeneous areas. In the suggested scheme, first create a binary image by choosing a suitable threshold value using Sharma-Mittal entropy. Window is applied on the binary image. locate all window coefficients equal to 1 except centre, centre equal to × as shown in Figure 3.

1	1	1
1	×	1
1	1	1

Figure 3: Window coefficients of 3×3 mask

Move the window on the whole binary image and find the probability of each central pixel of image under the window. Furthermore, the entropy of each Central Pixel of image under the window is calculated as $H(CP) = -p_c\ \ln(p_c)$.

Table 1: p and H of central pixel under window

p	1/9	2/9	3/9	4/9	5/9	6/9	7/9	8/9
H	0.2441	0.3342	0.3662	0.3604	0.3265	0.2703	0.1955	0.1047

where, p_c is the probability of central pixel CP of binary image under the window. When the probability of central pixel $p_c = 1$ then the entropy of this pixel is zero. Therefore, if the gray level of all pixels under the window homogeneous, then $p_c = 1$ and $\boldsymbol{H} = 0$. So, the central pixel is not an edge pixel. Other probabilities of entropy of central pixel under window are shown in Table 1.

In cases $p_c = 7/9$ and $p_c = 8/9$, the diversity for gray level of pixels under the window is low. Thus, in these cases, central pixel is not an edge pixel. In remaining cases, $p_c \leq 6/9$, the diversity for gray level of pixels under the window is high. Then, for these cases, central pixel is an edge pixel. Therefore, the central pixel with entropy greater than or equal to 0.244 is an edge pixel, otherwise not.

The following Algorithm describe the proposed technique for calculating the optimal threshold values and the edge detector.

Algorithm 2: Edge Detection

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1. Input: A grayscale image I of size $M \times N$ and t^{opt} , that has been calculated from algorithm 1.

2. Create a binary image: For all x, y,

if $I(x,y) \leq t^{opt}$ *then* $f(x,y) = 0$ *else* $f(x,y) = 1$.

3. Create a mask w of order $m \times n$, in our case ($m = 3, n = 3$)

4. Create an $M \times N$ output image g: For all x and y, Set $g(x,y) = f(x,y)$.

5. Checking for edge pixels:

Calculate: $a = (m-1)/2$ and $b = (n-1)/2$.

For all $y \in \{b+1, \dots, N-b\}$, and $x \in \{a+1, \dots, M-a\}$,

$sum = 0$;

For all $l \in \{-b, \dots, b\}$, and $j \in \{-a, \dots, a\}$,

if ($f(x,y) = f(x+j, y+l)$) *then* $sum = sum + 1$.

if ($sum > 6$) *then* $g(x,y) = 0$ *else* $g(x,y) = 1$.

6. **Output**: The edge detection image g of I.

Now, we can summarize the steps of our proposed technique as follows:

1.Find global threshold value (t_1) using Sharma-Mittal entropy. The image is segmented by t_1 into two parts, the object (Part1) and the background (Part2).

2.By using Sharma-Mittal entropy, we can select the locals threshold values (t_2) and (t_3) for Part1 and Part2, respectively.

3.Applying Edge Detection Procedure with threshold values t_1, t_2 and t_3.

4.Merge the resultant images of step 3 in final output edge image.

5. Experimental Results

To prove the efficiency of the suggested method, the algorithm is tested over a number of different grayscale images and compared with classical techniques. The performance of the approach is tested under noisy condition (Salt and Pepper noise) on test images. The images are corrupted via Salt & Pepper noise with 5%, 15% and 30% noise density before processing. The images detected by Canny, Sobel, Prewitt, LOG and the suggested method, respectively. All the concerned experiments were implemented on Intel® Core™ i3 2.10GHz with 4 GB RAM using MATLAB R2007b.

The proposed approach used the good characters of Sharma-Mittal entropy, to calculate the global and local threshold values. Hence, we ensure that the suggested scheme done better than the classical techniques.

In order to validate the results, we have used MATLAB to run the Canny, Sobel, Prewitt and LOG methods and the suggested algorithm 10 times for each image with different sizes. As shown in Figure 5, the suggested edge detector works effectively for different Grayscale images as compare to the run time of the classical methods.

Some chosen results of edge detections for these test images using the traditional techniques and proposed approach are shown in Figures 6 -9.

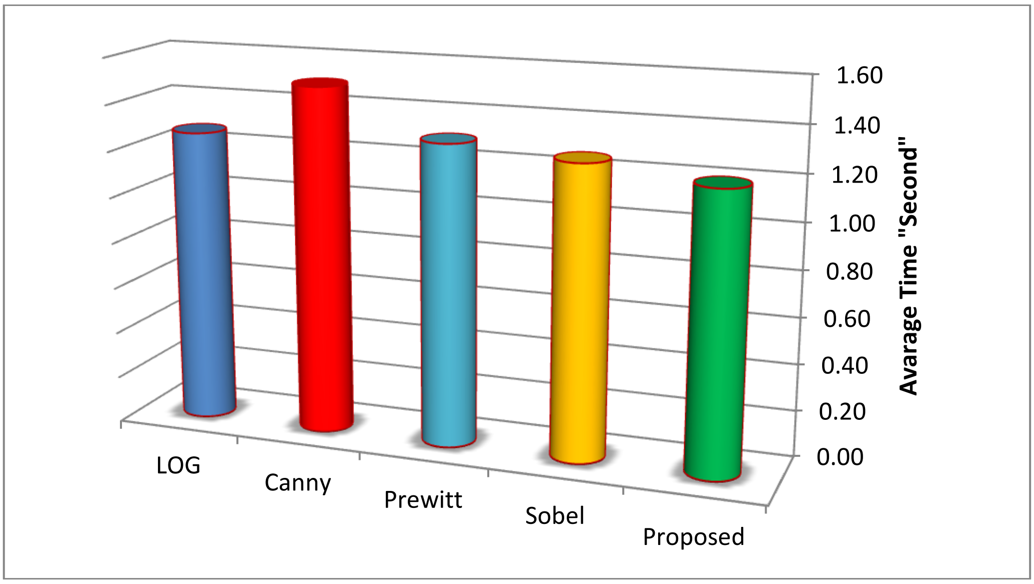
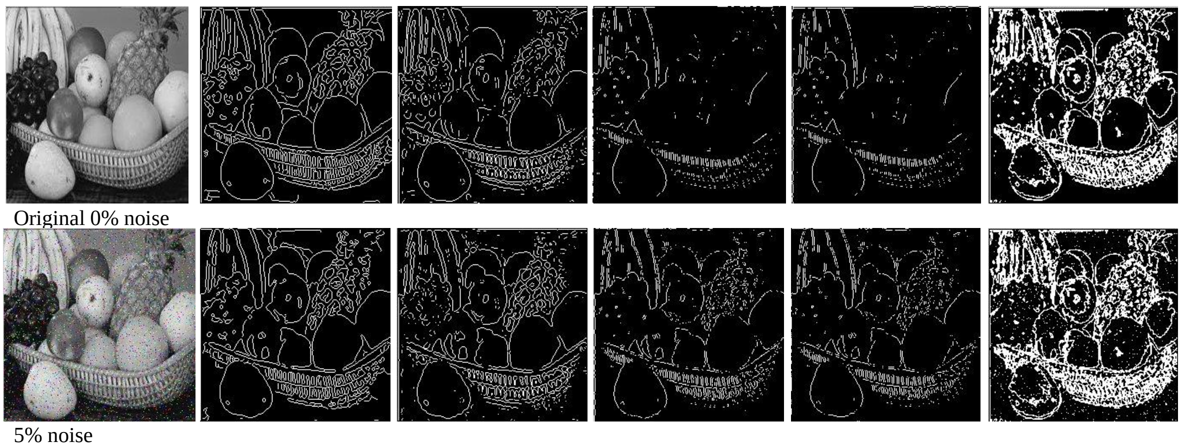
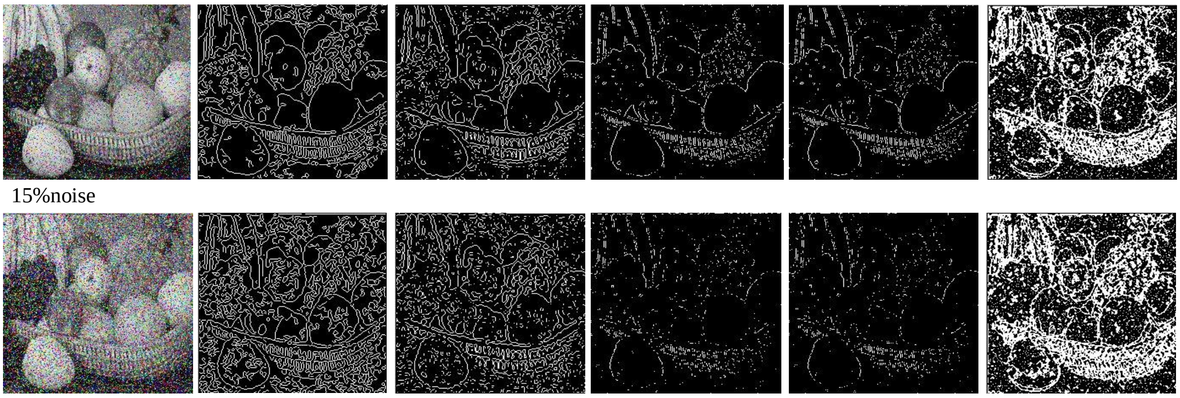


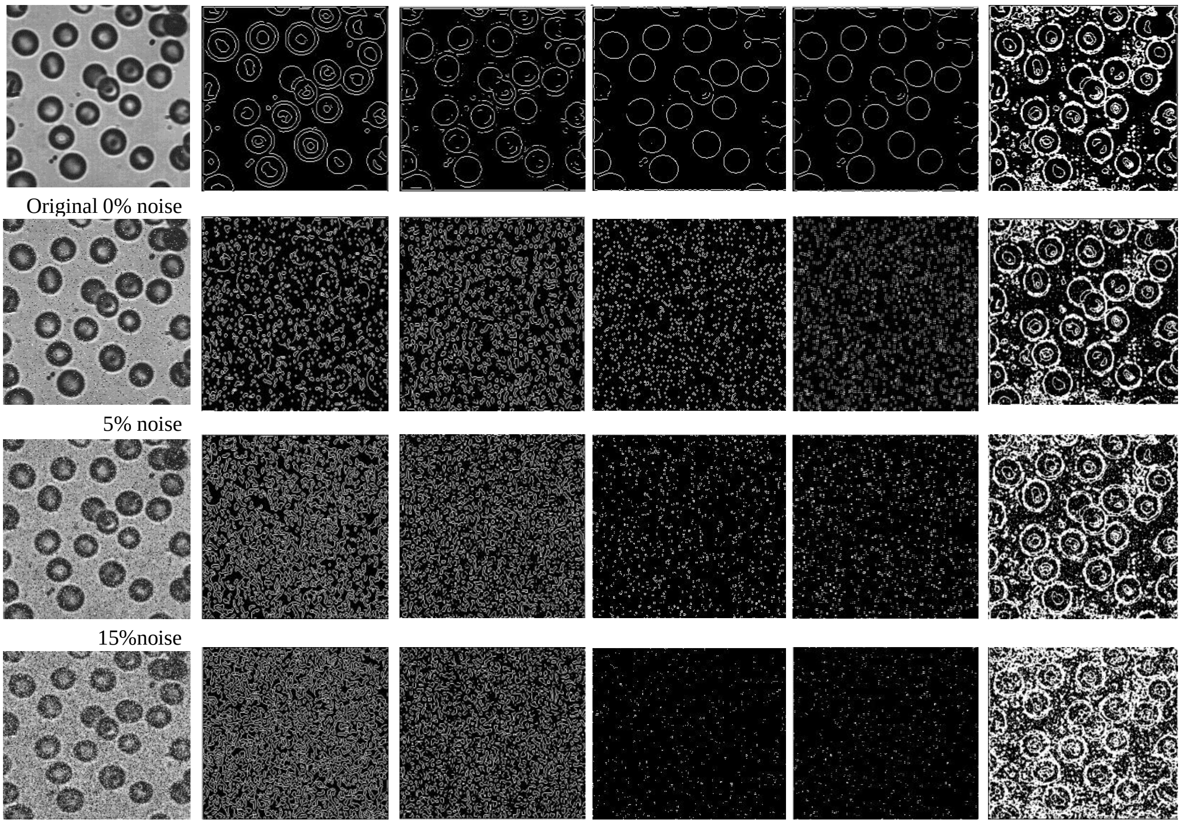
Figure 5: Chart time for proposed method and traditional methods with 512×512 pixel test images





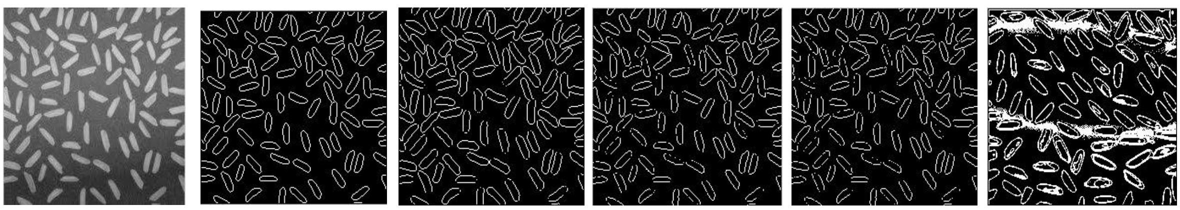
30% noise | Canny method | LOG method | Sobel method | Prewitt method | Proposed method α=0.4, β=0.8

Figure 6: Performance of Proposed Edge Detector for Fruits image with Various salt and pepper



30% noise | Canny method | LOG method | Sobel method | Prewitt method | Proposed method α=0.8, β=0.6

Figure 7: Performance of Proposed Edge Detector for Blood cells image with Various salt and pepper noise



Original 0% noise

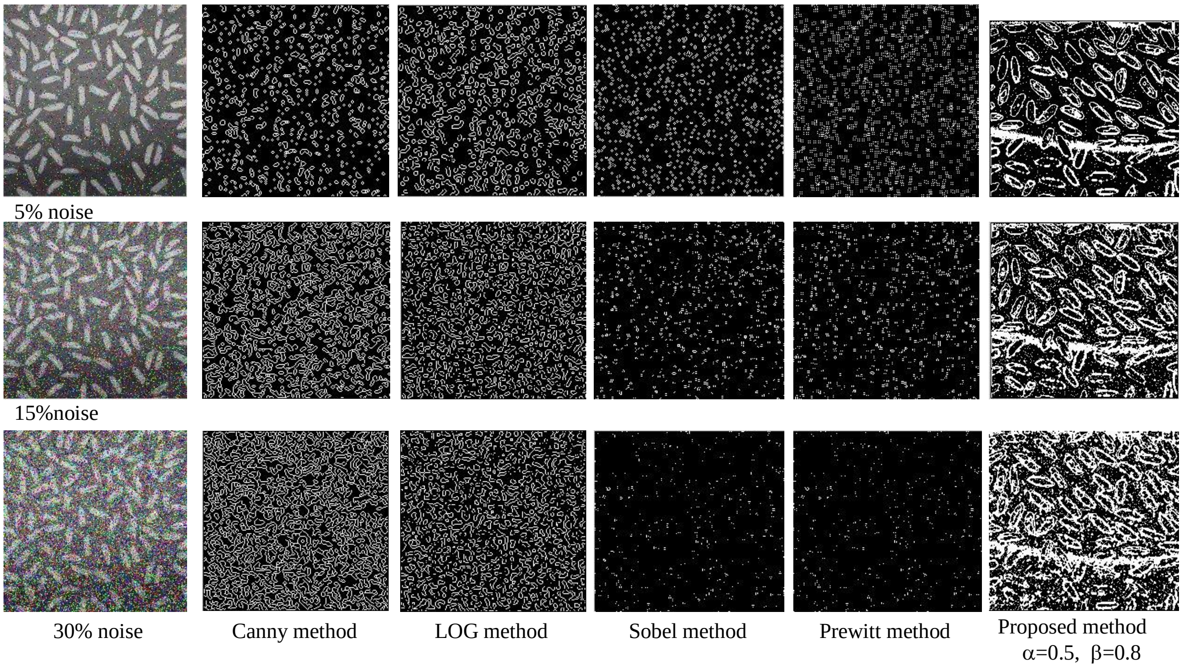


Figure 8: Performance of Proposed Edge Detector for seeds image with Various salt and pepper noise

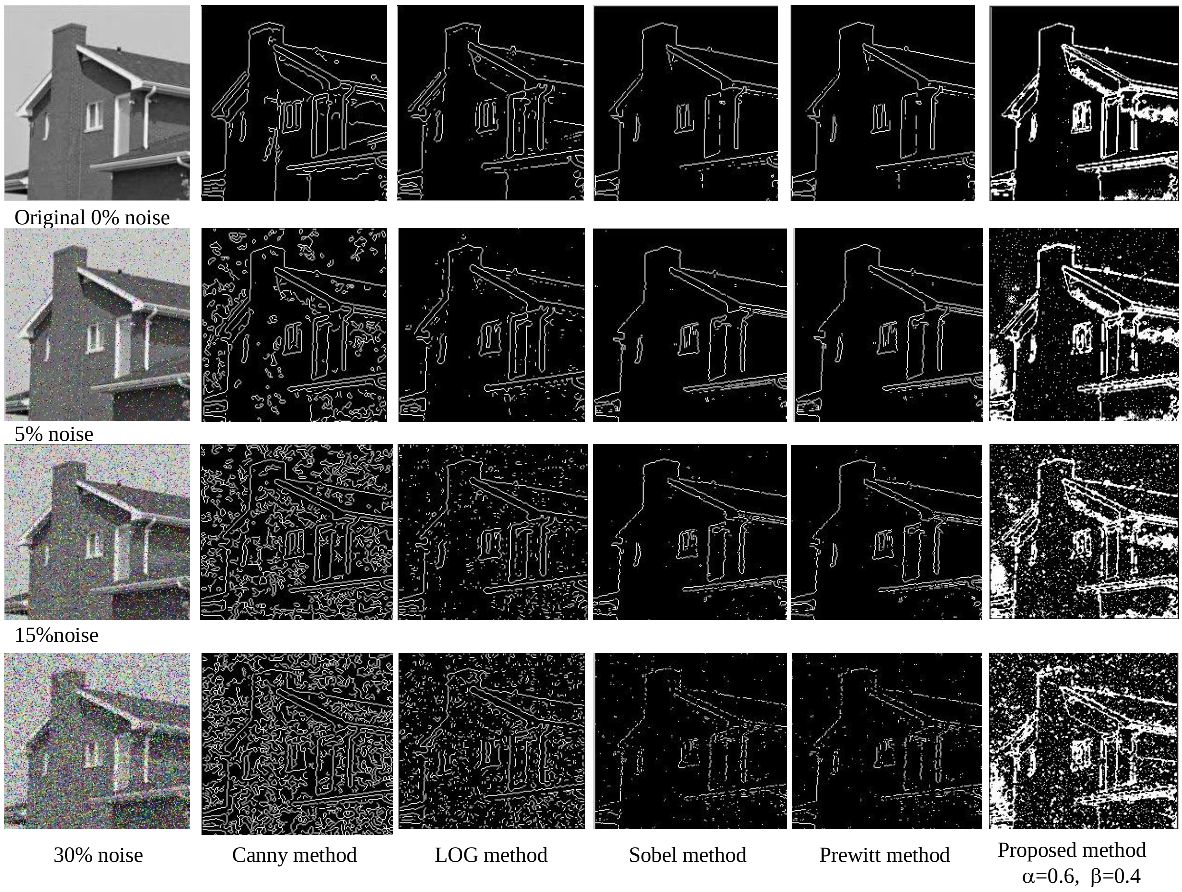


Figure 9: Performance of Proposed Edge Detector for House image with Various salt and pepper noise

6. Conclusion

In this paper, we presented a new approach for image edge detection based on generalized (α, β) entropy. The suggested approach has achieved the task of edge detection in a novel way. This method has been shown to provide good results in most cases and perform well when applied to noisy images. The experimental results show that using generalized Sharma-Mittal formalism of entropy is more viable than using classical methods in image edge detection. The main advantages of the method are its tolerance to image noise and its high rapidity.

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