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A Fuzzy Model for Measuring the Student Learning Based on Bloom's Taxonomy

[1]Michael Gr. Voskoglou, [2]Igor Ya. Subbotin

[1]Department of Mathematical Sciences, School of Technological Applications, Graduate Technological Educational Institute (T. E. I.) of Western Greece, Patras, Greece

[2]Department of Mathematics and Natural Sciences, College of Letters and Sciences, National University, Los Angeles, California, USA

mvosk@hol.gr, isubboti@nu.edu

Abstract

In the present paper we develop an improved version of the Triangular Fuzzy Assessment Model (TFAM) and we apply it to evaluate the students' progress in learning the topic "Real numbers" with respect to the principles of the Bloom's Taxonomy. The TFAM is a new original variation of the Center of Gravity (COG) defuzzification technique, which has been properly adapted in earlier papers to be used as an assessment tool. The central idea of TFAM is the replacement of the rectangles appearing in the graph of the membership function of the COG technique by isosceles triangles sharing common parts. In this way one treats better the ambiguous cases of student scores being at the boundaries between two successive assessment grades. Our model is validated by comparing it with traditional assessment methods (calculation of the means and GPA index), based on principles of the bivalent logic.

Keywords: *Bloom's taxonomy, Real numbers, Fuzzy Logic, Center of Gravity (COG) defuzzification technique, Triangular Fuzzy Assessment Model (TFAM).*

1. Introduction

The situations in which definitions have not clear boundaries are often appearing in everyday life. For example, it happens when we speak about the "high mountains" of a country, the "good players" of a football team, etc. The fuzzy sets theory was created in order to have a mathematical representation of such kind of situations.

Let U denote the universal set of the discourse. Then a *fuzzy set* A in U (or otherwise a fuzzy subset of U), initiated by Zadeh in 1965 [1], is defined in terms of the *membership function* m_A that assigns to each element of U a real value from the interval [0,1]. In more specific terms a fuzzy set A in U can be written as a set of ordered pairs in the form A = $\{(x, m_A(x)): x \in U\}$*, where $m_A : U \rightarrow [0,1]$.

*Notice that there are also alternative methods in use for the symbolic representation of a fuzzy set. In fact, if U is a finite set then A can be written as a symbolic sum in the form $\sum_{x\in U} m_A(x)/x$, if U is a denumerable set then A can be written as a symbolic infinite sum in the form $\sum_{i=0}^{\infty} m_A(x_i)/x_i$, while if U has the power of the continuous then A is usually written as a symbolic integral in the form $\int_U m_A(x)dx$.

The value $m_A(x)$, called the *membership degree (or grade) of x in A*, expresses the degree to which x verifies the characteristic property of A. Thus, the nearer is the value $m_A(x)$ to 1, the higher is the membership degree of x in A. The methods of choosing the proper membership function are empiric, based usually on statistical data of experiments performed with samples of the population under study. However, a necessary condition for the creditability of a fuzzy set in representing a real situation is that the criteria of the choice of the corresponding membership function are compatible to the common logic.

Obviously each classical (crisp) subset A of U can be considered as a fuzzy subset of U, with $m_A(x)=1$ if $x \in U$ and $m_A(x)=0$ if $x \notin U$. Most of the concepts of classical (crisp) sets can be extended in terms of the above definition to fuzzy sets.

Despite the fact that both operate over the same numeric range [0, 1], fuzzy set theory is distinct from probability theory. For example, the probabilistic approach yields the natural language statement "*there is an 85% chance that Mary is tall*", while the fuzzy terminology corresponds to the expression "*Mary's degree of membership within the set of tall people is 0.85*". The semantic difference is significant: The first view supposes that Mary is or is not tall (still caught in the law of the Excluded Middle); it is just that we only have a 85% chance of knowing in which set she is in. In contrast, fuzzy terminology supposes that Mary is "more or less" tall, or some other term corresponding to the value of 0.85. For general facts on fuzzy sets we refer to the book [2].

Fuzzy logic, the development of which is based on fuzzy sets theory, provides a rich and meaningful addition to standard Boolean logic. Unlike Boolean logic, which has only two states, true or false, fuzzy logic deals with truth values which range continuously from 0 to 1. Thus something could be *half true* 0.5 or *very likely true* 0.9 or *probably not true* 0.1, etc. In this way fuzzy logic allows one to express knowledge in a rule format that is close to a natural language expression and therefore it opens the door to construction of mathematical solutions of computational problems which are inherently imprecisely defined. New operations for the calculus of logic were also proposed and fuzzy logic showed to be in principle at least a generalization of classic logic [1, 3].

The methods of assessing the individual skills usually applied in practice are based on principles of the bivalent logic (yes-no). However, these methods are not probably the most suitable ones in ambiguous cases characterized by a degree of uncertainty. In Education, for example, the teacher is frequently not absolutely sure about a particular numerical grade characterizing a student's performance. Fuzzy logic, due to its nature of including multiple values, offers a wider and richer field of resources for this purpose.

In this paper we develop a fuzzy model for assessing the student success for learning with accordance to Bloom's taxonomy. This taxonomy, which has been applied in the USA by generations of teachers and college instructors in the teaching process [4], refers to a classification of the different learning objectives serving as a way of distinguishing the fundamental questions within the educational system.

The rest of the paper is organized as follows: In Section 2 we present the fundamentals of the Bloom's taxonomy. In Section 3 we develop our fuzzy model. In Section 4 we present an application of this model connected to the teaching of the real numbers at an intoductory College level. Finally, Section 5 is devoted to our final conlusions and a short discussion on future perspectives of research on this subject.

2. The Bloom's taxonomy

In 1956 Benjamin Bloom with collaborators Max Englehart, Edward Furst, Walter Hill, and David Krathwohl published a framework for categorizing educational goals, the *Taxonomy of Educational Objectives* [5][†]. Although named after Bloom, the publication of the taxonomy followed a series of conferences from 1949 to 1953, which were designed to improve communication between educators on the design of curricula and examinations. A revised version of the taxonomy was created in 2000 by Lorin Anderson [6], former student of Bloom. Since the taxonomy reflects different forms of thinking and thinking is an active process, in the revised version the names of its six major levels were changed from *noun* to *verb* forms. The six major levels of the revised taxonomy are presented in Figure 1, taken from [7].

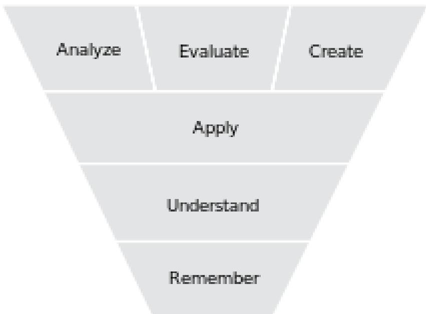

Figure 1: The six major levels of the Bloom's taxonomy

The above six levels in the taxonomy, moving through the lowest order processes to the highest, could be described as follows :

- *Knowing - Remembering:* Retrieving, recognizing, and recalling relevant knowledge from long-term memory. eg. find out, learn terms, facts, methods, procedures, concepts
- *Organizing - Understanding:* Constructing meaning from oral, written, and graphic messages through interpreting, exemplifying, classifying, summarizing, inferring, comparing, and explaining. Understand uses and implications of terms, facts, methods, procedures, concepts.
- *Applying:* Carrying out or using a procedure through executing, or implementing. Make use of, apply practice theory, solve problems, use information in new situations.
- *Analyzing:* Breaking material into constituent parts, determining how the parts relate to one another and to an overall structure or purpose through differentiating, organizing, and attributing. Take concepts apart, break them down, analyze structure, recognize assumptions and poor logic, evaluate relevancy.

[†] Bloom's taxonomy divides educational objectives into three domains: *cognitive* , *affective* and *psychomotor*, sometimes loosely described as "knowing/head", "feeling/heart" and "doing/hands" respectively. The volume published in 1956 [5] and the revision followed in 2000 [6] concern the cognitive domain, while a second volume published in 1965 on the affective domain. A third volume was planned on the psychomotor domain, but it was never published. However, other authors published their own taxonomies on the last domain. More details can be found in [7].

- *Generating - Evaluating:* Making judgments based on criteria and standards through checking and critiquing. Set standards, judge using standards, evidence, rubrics, accept or reject on basis of criteria.
- *Integrating - Creating:* Putting elements together to form a coherent or functional whole; reorganizing elements into a new pattern or structure through generating, planning, or producing. Put things together; bring together various parts; write theme, present speech, plan experiment, put information together in a new & creative way

Most reserchers and educators consider the last three levels --analyzing, evaluating and creating – as being parallel. It is obvious that using Bloom's higher levels helps the students become better problem solvers.

Teaching a topic, the teacher should arrange his/her class work in the order to synchronize it with these six steps of Blom’s Taxonomy. The typical questions for evaluating the student achievement at the corresponding level are the following:

Knowing questions focus on clarifying, recalling, naming, and listing:
Which illustrates...?
Write... in standard form....
What is the correct way to write the number of... in word form?
Organizing questions focus on arranging information, comparing similarities/ differences, classifying, and sequencing:
Which shows... in order from...?
What is the order...?
Which is the difference between a... and a...?
Which is the same as...?
Express... as a...?
Applying questions focus on prior knowledge to solve a problem:
What was the total...?
What is the value of...?
How many... would be needed for...?
Solve....Add/subtract....Find....Evaluate....Estimate....Graph....
Analyzing questions focus on examining parts, identifying attributes/ relationships /patterns, and main idea:
Which tells...?
If the pattern continues,....
Which could...?
What rule explains/completes... this pattern?
What is/are missing?
What is the best estimate for...?
Which shows...?
What is the effect of...?
Generating questions focus on producing new information, inferring, predicting, and elaborating with details:
What number does... stand for?
What is the probability...?
What are the chances...?
What effect...?

Integrating questions focus on connecting/combining/summarizing information, and restructuring existing information to incorporate new information:
How many different...?
What happens to... when...?
What is the significance of...?
How many different combinations...?
Find the number of..., ..., and ... in the figure below.
Evaluating questions focus on reasonableness and quality of ideas, criteria for making judgments and confirming accuracy of claims:
Which most accurately...?
Which is correct?
Which statement about... is true?
What are the chances...?
Which would best...?
Which would... the same...?
Which statement is sufficient to proven...?

Bloom's taxonomy serves as the backbone of many teaching philosophies, in particular those that lean more towards skills rather than content. The emphasis on higher-order thinking inherent in such philosophies is based on the top levels of the taxonomy including analysis, evaluation, synthesis and creation. Bloom's taxonomy can be used as a teaching tool to help balance assessment and evaluative questions in class, assignments and texts to ensure all orders of thinking are exercised in student's learning.

3. The fuzzy assessment model

Reasoning with fuzzy rules is a forward-chaining procedure. The initial numeric data values are *fuzzified*, that is, turned into fuzzy values using the membership functions. Instead of a match and conflict resolution phase where we select a triggered rule to fire, in fuzzy systems, all rules are evaluated, because all fuzzy rules can be true to some degree ranging from 0 to 1. The antecedent clause truth values are combined using fuzzy logic operators. Next, the fuzzy sets specified in the consequent clauses of all rules are combined using the rule truth values as scaling factors. The result is a single fuzzy set, which is then *defuzzified* to return a crisp output value.

There are several defuzzification techniques in use, the most popular being probably the *centre of gravity* (COG) method [8]. According to this method the fuzzy data is represented by the coordinates of the COG of the level's section contained between the graph of the membership function involved and the OX axis.

In earlier papers ([9-12], etc) the GOG technique has been properly adapted by the authors of this paper to be used as an assessment method of individual skills. Here we shall apply an improved form of a recently developed [13] variation of the above assessment method that we have called *Triangular Fuzzy Assessment Model* (TFAM).

Let G a student group participating in a certain activity (learning, problem-solving, etc) and let A, B, C, D and F be the linguistic labels of excellent, very good, good, fair and unsatisfactory performance respectively with respect to this activity.

Set U = {A, B, C, D, F}. Then G can be expressed as a fuzzy set in U in the form G= {(x, m(x)): $x \in U$}, where y=m(x) is the corresponding membership function. The main idea of TFAM is the replacement of the rectangles appearing in the graph of the COG technique (e.g. see Figure 1 of [11]) by triangles. Therefore, we shall have five such triangles in the resulting scheme, each one corresponding to a students' grade (F, D, C, B and A respectively). Without loss of generality and for making our calculations easier we consider isosceles triangles with bases of length 10 units lying on the OX axis. The height to the base of each triangle is equal to the percentage of students' of the group under assessment who achieved the corresponding grade. We allow for any two adjacent triangles to have 30% of their bases belonging to both of them. In this way, we treat better the existing uncertainty situations of marginal students' scores, being at the boundaries between two successive grades[‡].

The resulting scheme is presented in Figure 2. The student group under assessment can be represented, as in the COG technique, as a fuzzy set in U, whose membership function $y = m(x)$ has as graph the line $OA_1B_1A_2B_2A_3$ B_3A_4 $B_4A_5C_9$. It is easy then to calculate the coordinates (b_{i1}, b_{i2}) of the points B_i, i = 1, 2, 3, 4, 5. In fact, B_1 is the intersection of the straight line segments A_1C_2 and C_1A_2, B_2 is the intersection of C_3A_3, and A_2C_4 and so on. Therefore, it is straightforward to determine the analytic form of $y=m(x)$ consisting of 10 branches, corresponding to the equations of the straight lines OA_1, A_1B_1, B_1A_2, A_2B_2, B_2A_3, A_3B_3, B_3A_4, A_4B_4, B_4A_5 and A_5C_9 in the intervals [0, 5), [5, b_{11}), [b_{11}, 12), [12, b_{12}), [b_{12}, 19), [19, b_{13}), [b_{13}, 26), [26. b_{14}), [b_{14}, 33) and [33, 38] respectively.

However, when applying the TFAM, the use of the analytic form of $y = m(x)$ is not needed, as it happens in the case of the classical COG technique, for the calculation of the COG of the resulting level's area. In fact, since the marginal cases of students' grades should be considered as common parts for any pair of the adjacent triangles, it is logical to don't subtract the areas of the intersections from the area of the corresponding level's section, although in this way we count them twice; e.g. placing the ambiguous cases B+ and A- in both regions B and A. In other words, the classical COG technique, which calculates the coordinates of the COG of the area between the graph of the membership function (line $OA_1B_1A_2B_2A_3$ B_3A_4 $B_4A_5C_9$) and the OX axis (see Figure 1), thus considering the areas of the "common" triangles $C_1B_1C_2$, $C_3B_2C_4$, $C_5B_3C_6$ and $C_7B_4C_8$ only once, is not the proper one to be applied in the above situation.

Indeed, in this case it is reasonable to represent each one of the five triangles OA_1C_2, $C_1A_2C_4$, $C_3A_3C_6$, $C_5A_4C_8$ and $C_7A_5C_9$ of Figure 2 by their COG's F_i, i=1, 2, 3, 4, 5 and to consider the entire level's area as the system of these points-centers. More explicitly, the steps of the whole construction of the TFAM are the following:

[‡] It is a very common approach in such cases to divide the interval of the specific grades in three parts and to assign the corresponding grade using + and - . For example, in a scale of scores from 0 to 100 we could have 75 – 77 = B-, 78 – 80 = B, 81 – 84 = B+. However, this consideration does not reflect the common situation, where the teacher is not sure about the grading of the students whose performance could be assessed as marginal between and close to two adjacent grades; for example, something like between 74 (C) and 75 (B). The TFAM fits to this kind of situations.

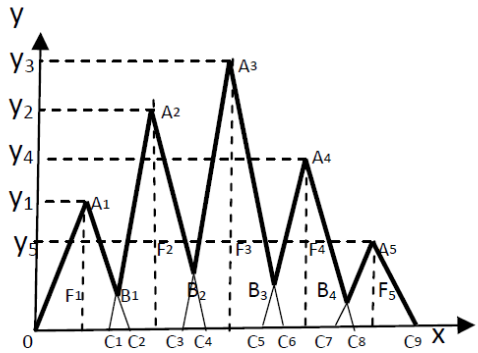

Figure 2: the membership function's graph of TFAM

1. Let y_1, y_2 , y_3, y_4, y_5 be the percentages of the students in the group getting F, D, C, B, and A grades respectively, then $y_1 + y_2 + y_3 + y_4 + y_5 = 1$ (100%).
2. We consider the isosceles triangles with bases having lengths of 10 units each and their heights y_1, y_2, y_3, y_4, y_5 in the way that has been illustrated in Figure 1. Each pair of adjacent triangles has common parts in the base with length 3 units.
3. We calculate the coordinates (x_{c_i}, y_{c_i}) of the COG F_i, i=1, 2, 3, 4, 5 of each triangle as follows: The COG of a triangle is the point of intersection of its medians, and since this point divides the median in proportion 2:1 from the vertex, we find, taking also into account that the triangles are isosceles, that $y_{c_i} = \frac{1}{3} y_i$. Further, since the triangles' bases have a length of 10 units, it is easy to observe that $x_{ci} = 7i\text{-}2$.
4. We consider the system of the centers F_i, i=1, 2, 3, 4, 5 and we calculate the coordinates (X_c, Y_c) of the COG F_c of the whole level's area considered in Figure 1 from the following formulas, derived from the commonly used in such cases definition [14]:

$$X_c = \frac{1}{S}\sum_{i=1}^{5} S_i x_{c_i} \text{ , } Y_c = \frac{1}{S}\sum_{i=1}^{5} S_i y_{c_i} \tag{1}$$

In formulas (1) S denotes the whole of the considered level's area and Si, i= 1, 2, 3, 4, 5 denote the areas of the corresponding triangles. Therefore $Si = 5y_i$ and $S=\sum_{i=1}^{5} S_i = 5\sum_{i=1}^{5} y_i = 5$. Thus, from formulas (1) we finally get:

$$X_c = \frac{1}{5}\sum_{i=1}^{5} 5y_i(7i-2) = 7\sum_{i=1}^{5} iy_i - 2$$

$$Y_c = \frac{1}{5}\sum_{i=1}^{5} 5y_i\left(\frac{1}{3}y_i\right) = \frac{1}{5}\sum_{i=1}^{5} y_i^2 \text{ .} \tag{2}$$

5. We determine the area where the COG F_c lies as follows : For i, j=1, 2, 3, 4, 5, we have that

$0 \le (y_i - y_j)^2 = y_i^2 + y_j^2 - 2y_i y_j$, therefore $y_i^2 + y_j^2 \ge 2y_i y_j$, with the equality holding if, and only if, $y_i = y_j$. Therefore $1 = (\sum_{i=1}^{5} y_i)^2 = \sum_{i=1}^{5} y_i^2 + 2\sum_{i,j=1, i\ne j}^{5} y_i y_j \le \sum_{i=1}^{5} y_i^2 + \sum_{i,j=1, i\ne j}^{5} (y_i^2 + y_j^2) = 5\sum_{i=1}^{5} y_i^2$, or $\sum_{i=1}^{5} y_i^2$

$\geq \frac{1}{5}$ (3), with the equality holding if, and only if, $y_1 = y_2 = y_3 = y_4 = y_5 = \frac{1}{5}$. In the case of equality the first of formulas (2) gives that $X_c = 7(\frac{1}{5} + \frac{2}{5} + \frac{3}{5} + \frac{4}{5} + \frac{5}{5}) - 2 = 19$. Further, combining the inequality (3) with the second of formulas (2) one finds that $Y_c \geq \frac{1}{25}$ Therefore the unique minimum for Y_c corresponds to the COG

F_m (19, $\frac{1}{25}$). The ideal case is when $y_1=y_2=y_3=y_4=0$ and $y_5=1$. Then from formulas (2) we get that $X_c = 33$ and $Y_c = \frac{1}{5}$.Therefore the COG in this case is the point F_i (33, $\frac{1}{5}$). On the other hand, the worst case is when $y_1=1$ and $y_2=y_3=y_4=y_5=0$. Then from formulas (2), we find that the COG is the point F_w (5, $\frac{1}{5}$). Therefore the "area" where the COG F_c lies is the triangle $F_w F_m F_i$ presented in Figure 3.

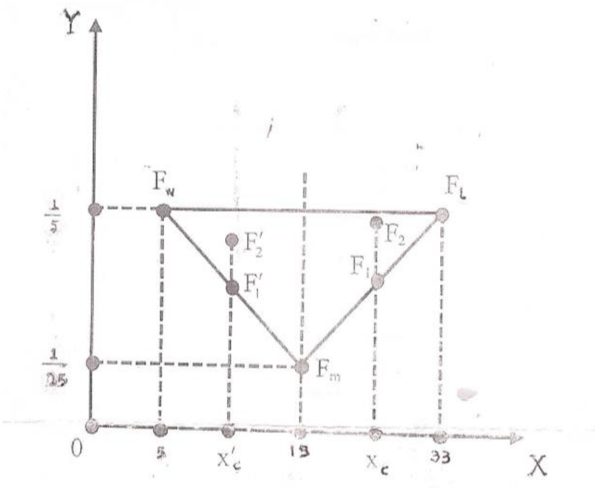

Figure 3: The area where the COG lies

6. We formulate our criterion for comparing the performance of two (or more) groups' as follows: From elementary geometric observations (see Figure 3) it follows that for two groups the group having the greater X_c performs better. Further, if the two groups have the same $X_c \geq 19$, then the group having the COG which is situated closer to *Fi* is the group with the greater Y_c. Also, if the two groups have the same $X_c < 19$, then the group having the COG which is situated farther to *Fw* is the group with the smaller Y_c. Based on the above considerations it is logical to formulate our criterion for comparing the two groups' performance in the following form:

- *Among two or more groups the group with the greater X_c performs better.*
- *If two or more groups have the same $X_c \geq 19$, then the group with the greater Y_c performs better.*
- *If two or more groups have the same $X_c < 19$, then the group with the lower Y_c performs better.*

As it becomes evident from the above presentation, the application of the TFAM is simple in practice needing no complicated calculations in its final step. Further, our criterion shows that the assessment of the student performance is based on the values of X_c. But, as it turns out from the first of formulas 2, calculating the value of X_c greater coefficients (weights) are assigned to the higher scores. Therefore TFAM provides a weighted measure focusing on the student *quality performance.*

4. An application on teaching the real numbers

4.1 Description

The following application was performed with subjects the students of two different departments (30 students in each department) of the School of Technological Applications (prospective engineers) of the Graduate Technological Educational Institute (T. E. I.) of Western Greece attending the common course "Mathematics I" of their first term of studies and having the same instructor. This course involves an introductory module repeating and extending the students' knowledge from secondary education about the real numbers. After the module was taught, the instructor wanted to investigate the students' progress according to the principles of the Bloom's Taxonomy. For this, he asked them to answer in the class the written test presented in the Appendix of this paper, which is divided in six different parts, one for each level of the Taxonomy. The students' answers were assessed separately for each level in a scale from 0 to 100 and the means obtained correspond to each student's overall performance.

4.2 Results

Denote by L_i, i=1, 2, 3, 4, 5, 6 the levels of Knowing-Remembering, Organizing-Understanding, Applying, Analyzing, Generating-Evaluating and Integrating- Creating respectively of the Bloom's Taxonomy and by P the student overall performance. Then the test's results are summarized in the following two tables:

Table 1: Results of the first department

Grade	L_1	L_2	L_3	L_4	L_5	L_6	P
A(85-100)	8	6	5	3	2	3	4
B(84-75)	9	11	10	8	7	8	9
C(74-60)	10	9	10	12	10	8	10
D(59-50)	3	3	3	5	7	8	5
F(<50)	0	1	2	2	4	3	2

Table 2: Results of the second department

Grade	L_1	L_2	L_3	L_4	L_5	L_6	P
A(85-100)	9	8	6	4	3	3	5
B(84-75)	6	7	9	7	7	6	8
C(74-60)	9	8	10	12	10	8	9
D(59-50)	6	7	4-	4	7	11	7
F(<50)	0	0	1	3	3	2	1

4.3 Evaluation of the results using the TFAM

From Table 1 we obtain the following percentages for the level L_1: $y_1=0$, $y_2=\frac{3}{30}$, $y_3=\frac{10}{30}$, $y_4=\frac{9}{30}$ and $y_5=\frac{8}{30}$. Therefore, applying the first of formulas (2) one finds that $X_c=7(\frac{6}{30}+\frac{30}{30}+\frac{36}{30}+\frac{40}{30})-2=\frac{724}{30}\approx 24.13$. Similarly one finds the following values of X_c: 23.2 for L_2, 20.87 for L_3, 20.17 for L_4, 18.07 for L_5, 19 for L_6 and 20.87 for the student overall performance P.

In the same way one finds from Table 2 the following values of X_c: 23.2 for L_1, 22.73 for L_2, 22.5 for L_3, 20.17 for L_4, 19 for L_5, 18.3 for L_6 and 21.1 for P.

On comparing the values of X_c for the two departments and according to the first case of the criterion stated in section 3 one concludes that the first department demonstrated a better performance at the levels L_1, L_2 and L_6 of the Bloom's Taxonomy, while the second department demonstrated a better performance at the levels L_3 and L_5. Further, the two departments demonstrated the same performance at the level L_4, while the second department demonstrated a better overall performance than the first one. In general, the overall performance of the two departments as well as their performance at each stage of the Bloom's Taxonomy can be characterized as more than satisfactory, since the corresponding values of X_c are in all cases greater than the half of its value in the ideal case, which is equal to $\frac{33}{2}=16.5$ (see Figure 3).

We also observe that the performance of each department is decreasing from level L_1 to level L_4, which was expected since the success at the higher levels is based on the lower levels. However, for the first department this does not happen for the last three levels, a fact which is compatible to the view that the three higher levels of the Taxonomy are parallel to each other (see section 2 – Figure 1).

4.4 Comparison of the TFAM with the traditional assessment methods

Most of the traditional assessment methods, which are based on the principles of the bivalent logic, measure the students' *mean performance*. Therefore, the conclusions obtained by applying these methods may differ from the conclusions obtained by applying the TFAM, which, as we have seen in section 3, measures the students' *quality performance* by assigning higher coefficients (weights) to the higher scores. For example, in the hypothetical case where the students of the last column of Table 1 obtained the highest scores of the corresponding grade (i.e. 4 students scored 100, 9 students scored 84, etc), while the students of the last column of Table 2 obtained the lowest scores of the corresponding grade (i.e. 5 students scored 85, 8 students scored 75, etc), calculating the means one finds an average score 64.51 for the first and 53.33 for the second department. Therefore, the first department demonstrates a much better mean overall performance than the second one, in contrast to their quality performance measured by TFAM.

A traditional assessment method - very popular in the USA- which measures the students' quality performance is the *Grade Point Average* (GPA) index. In terms of the student percentages the GPA index is calculated by the formula [15]:

$$\text{GPA} = y_2+2y_3+3y_4+4y_5 \tag{4}$$

In the worst case (y_1=1 and y_2=y_3=y_4=y_5=0) formula (4) gives that GPA=0, while in the ideal case (y_1=y_2=y_3=y_4=0 and y_5=1) it gives that GPA=4. Therefore we have that $0 \leq \text{GPA} \leq 4$.

Applying (4) on the data of the first column of Table 1 one finds that GPA= $\frac{3}{30}+\frac{20}{30}+\frac{27}{30}+\frac{32}{30} \approx 2.73$ at level L_1 of the Taxonomy for the first department. Similarly one finds the GPA values 2.6 for L_2, 2.43 for L_3, 2.17 for L_4, 1.87 for L_5, 2 for L_6 and 2.17 for the overall performance of the first department. In the same way working with the data of Table 2 one finds the GPA values 2.6, 2.53, 2.5, 2.17, 2, 1.9 and 2.3 respectively for the second department. Therefore, the two departments demonstrate the same performance at level L_4, the first department demonstrates a better performance at levels L_1, L_2 and L_6, while the second department demonstrates a better performance at levels L_3, L_5 and a better overall performance than the first department. These findings agree with the corresponding ones obtained by applying the TFAM. However, according to the GPA index the performance of the first department at level L_5 and of the second department at level L_6 were found to be less than satisfactory, since their GPA values are smaller than the half of its ideal value, which is equal to 2. This difference is due to the fact that, as it can be easily observed on comparing formula (4) with the first of formulas (2), the TFAM assigns greater weights and therefore it is *more sensitive* than the GPA index *to the higher scores*.

5. Conclusion

In the present paper we developed an improved version of the Triangular Fuzzy Assessment Model (TFAM) and we applied it to evaluate the students' progress for learning the real numbers with respect to the principles of the Bloom's Taxonomy. In the design of the TFAM the rectangles appearing in the graph of the membership function of the COG technique were replaced by isosceles triangles sharing common parts. In this way one treats better the ambiguous cases of student scores being at the boundaries between two successive assessment grades. Our model was validated by comparing it with traditional assessment methods (calculation of the means and GPA index), based on principles of the bivalent logic.

Our future plans include the application of the same model for studying the students' progress with respect to the principles of the Bloom's Taxonomy in other fields of knowledge (not only for mathematics). Also, since the TFAM seems to have the potential of a general assessment method, our research perspectives focus on applying it to evaluate other kind of human activities in Science, games, decision making, etc.

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Appendix: The test used in our application

Topic: Real numbers (In introductory College level)

Questions

1. Knowing - Remembering:

Give the definitions and examples of a periodic decimal and of an irrational number (in the form of an infinite decimal).

2. Organizing:

Compare the set of all fractions with the set of periodic decimals. Compare the set of irrational numbers with the set of all roots (of any order) that have no exact values.

3. Applying:

Which of the following numbers are natural, integers, rational, irrational and real numbers?

-2 , $-\frac{5}{3}$, 0 , 9.08 , 5 , $7.333...$, $\pi = 3.14159...$, $\sqrt{3}$, $-\sqrt{4}$, $\frac{22}{11}$, $5\sqrt{3}$, $-\frac{\sqrt{5}}{\sqrt{20}}$,

$\left(\sqrt{3}+2\right)\left(\sqrt{3}-2\right)$, $-\frac{\sqrt{5}}{2}$, $\sqrt{7}-2$, $\sqrt{\left(\frac{5}{3}\right)^2}$

4. Analyzing:

Find the digit which is in the 1005th place of the decimal 2.825342342......

Write the number 0.345345345... in the fractional form.

Compare the numbers 5 and 4.9999…

Construct the line segment of length $\sqrt{3}$ with the help of the Pythagorean Theorem. Give a geometric interpretation.

5. Generating- Evaluating:

Justify why the decimals 2.00131311311131111..., 0.1234567891011... are irrational numbers.

Construct the line segment of length $\sqrt[3]{2}$ by using the graph of the function f(x)= $\sqrt[3]{x}$

6. Integrating- Creating:

Define the set of the real numbers in terms of their decimal representations (this definition was not given by the instructor in the class before the test).