Use of Grey Numbers for Assessing the Effect of Application of the Flipped Learning Model on the Performance of a Mathematics Class

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Abstract

Flipped teaching is an innovative methodology developed during the last years and requiring that the acquisition of the new knowledge is done outside the classroom through the use of computers and of other technological tools. On the contrary, what is traditionally taken as homework is done in the class under the instructor's supervision in order to favor the student autonomy and increase the time devoted to problem solving and deepening of the content. Flipped teaching is highly based on the principles of social constructivism for learning. The present work describes an experiment on flipped teaching of mathematics (Conic Sections) performed in a university classroom in Greece, where the student assessment was done by calculating the GPA index (quality performance) and by using grey numbers (mean performance). The latter was necessary since qualitative and not numerical grades were used for the student assessment

Keywords: *Social Constructivism, Flipped Learning (FL), Mathematics Education, Grey System Theory, Grey Numbers (GN's), Grade Point Average (GPA) index..*

1. Introduction

The *constructivist view* and the *socio-cultural theories for learning* have been recently highly influential in addressing mathematical knowledge and the learning of mathematics. The constructivist view involves two principles: Knowledge is actively constructed by the learner, not passively received from the environment, and second the importance of the "*coming to know",* which is a process of adaptation based on and constantly modified by the learner's experience of the world (Von Glasersfeld [1]). Further, according to the socio-cultural approach learning takes place within some socio-cultural setting. Shared meanings are formed through negotiation in the learning environment, leading to the development of common knowledge (Wenger [2]; Jaworski [3]). The combination of constructivism with the socio-cultural ideas is known as *social constructivism* (Driver et al. [4]; Ernest [5]).

It is difficult to deny that in our modern society of knowledge and information computers are a valuable tool for teaching and learning. The wealth of information in hands of students, the animation of figures and representations provided by the proper educational software that increases the students' imagination and problem solving skills, the rich variety of data and resources that teachers can use working with their students to keep them engaged in the classroom, etc. are some of the benefits obtained by using the computers in Education.

An innovative teaching methodology has been promoted with the help of computers during the last few years known as *flipped learning* (FL) or *reverse learning* which has its roots to the work of Lage, Piatt and Tregla [6]. At the educational level, FL is considered as a mixed process that involves both online and face-to-face teaching and requires turning around the daily didactic processes to which we are accustomed. In fact, the acquisition of the new knowledge by the student is done outside the classroom through the use of digital platforms and technological tools that the teacher or other specialists have developed. Sams and Bergmann [7] were able to develop an online audiovisual teaching material so that students could do it regardless of factors such as place and time. On the contrary, what is traditionally undertaken as homework is done in class with the supervision of the teacher in order to favor the adequacy of learning and autonomy of students and increase the time devoted to practice, problem solving and deepening of content (Lee et al. [8]). FL is highly based on the ideas of *social constructivism*.

Although social constructivism and more recently FL have become very popular in school education for teaching mathematics (Voskoglou [9]), in the university departments of positive sciences the majority of the instructors still prefer the traditional way of the teacher-centred mathematics instruction. However, recent research results (Lahdenpera et al. [10]; Syh-Jong Jang [11]; etc.) suggest that the application of methods based on ideas of the social constructivism could offer many advantages to the teaching and learning of mathematics at university level.

Those findings gave us the impulsion to perform the present experimental research concerning the application of FL for teaching mathematics to fresher students (prospective engineers) of the Graduate TEI of Western Greece. The rest of the article is organized as follows: In Section 2 methods are described to calculate the mean performance of a student group when using linguistic grades. In Section 3 the elements of the theory of Grey numbers are exposed which are necessary for the understanding of the rest of the paper. In Section 4 the assessment method using Grey Numbers is developed and in Section 5 a classroom experiment on flipped learning is described and its outcomes are evaluated with the help of the GPA index (quality performance) and the grey numbers (mean performance). The article closes with the general conclusions presented in Section 6.

2. Methods for Assessing the Mean Performance of a Student Group

The student assessment, being in general a societal requirement, is a very important part of the educational process that enables students and their parents to get an idea about their progress, and helps teachers to evaluate their teaching methods and to reorganize their teaching plans accordingly.

When the student performance is evaluated with numerical scores, then the traditional way to assess the *mean performance* of a student class is the calculation of the average of those scores. However, either for reasons of more elasticity or to comfort the teacher's existing uncertainty about the exact value of the numerical scores corresponding to each student's performance, frequently in practice the assessment is made not by numerical scores but by *linguistic grades*, like excellent, very good, good, etc. This involves a degree of vagueness and makes the calculation of the mean value of the student grades impossible. A popular in such cases method for evaluating the overall performance of a student class is the calculation of the *Grade Point Average (GPA) index* (e.g. see Voskoglou [12], Chapter 6, p.125). However, GPA is a weighted average in which greater coefficients (weights) are assigned to the higher grades. That means that GPA reflects not the mean, as we have in mind, but the *quality performance* of a student class.

In an effort to estimate the mean student performance in such vague assessment cases we have used in earlier works tools from *Fuzzy Logic*. More explicitly, representing the student class as a fuzzy set in the set of the linguistic grades used for the student assessment, we calculated the existing in it *uncertainty* **(**probabilistic or possibilistic)**.** This approach is based on the classical principle of information theory that the greater is the reduction of the uncertainty, the more the new information obtained by the class and therefore the better the student performance (e.g. see Chapter 5 of [12]). Nevertheless, this method has two disadvantages. First it involves laborious calculations and second it can be used for comparing the performance of two different classes only under the assumption that they have been proved to be of the same potential before the corresponding activity (e.g. test, problem-solving, learning a new subject matter, etc.), in the sense that the existing in both of them uncertainty is the same. However, that condition does not always hold in real situations. For this reason we have used in a later work *Triangular Fuzzy Numbers (TFNs)* for assessing the student mean performance (e.g. see Chapter 7 of [12]). This method has been proved to be simpler in its application and more accurate than the calculation of the uncertainty.

In the present paper an alternative method will be developed for estimating the student mean performance in such vague situation by using *Grey Numbers (GNs)* instead of TFNs. Although the above two methods (GNs and TFNs) have been proved to be equivalent (Voskoglou and Theodorou [13], Section 3), the use of the GNs reduces significantly the required computational burden.

3. Grey Numbers

Frequently in the everyday life as well as in many applications of science and engineering a system's data cannot be easily determined precisely and in practice estimates of them are used. The reason for this is that in large and complex systems, like the socio – economic, the biological ones, etc., many different and constantly changing factors are usually involved, the relationships among which are indeterminate, making their operation mechanisms to be not clear.

Nowadays two are the main tools for handling such approximate data: *Fuzzy Logic (FL)*, which is based on the notion of *Fuzzy Set (FS)* initiated by Zadeh [14] in 1965 and the theory of *Grey System (GS)* initiated by Deng [15] in 1982. The GS theory has been mainly developed in China and finds today important applications in agriculture, economy, management, industry, ecology, environment, meteorology, geography, geology, earthquakes, history, military affairs, sports, traffic, material science, biological protection and in many other fields of the human activity (see Deng [16] and its relevant references). Roughly speaking, a GS is understood to be any investigated object with "poor" information. More explicitly, the systems which lack information, such as structure message, operation mechanism and behaviour document, are referred to as GS's. For example, the human body, agriculture, economy, etc., are GS's. Usually, on the grounds of existing grey relations and elements one can identify where "grey" means poor, incomplete, uncertain, etc.

The aim of the GS theory is to provide techniques, notions and ideas for analyzing latent and intricate systems, including the establishment of non-function models, the development of a grey process replacing an existing stochastic process, the transformation of disorderly raw data into a more regular series by grey generating techniques instead of modelling with the original data; grey decision making, grey forecasting control replacing classical control, the study of feeling and emotion functions and fields with whitening functions, the study in general of grey mathematics instead of classical mathematics, etc. (for more details see Deng [16]).

An effective tool for handling the approximate data of a GS is the use of GN's. A GN is an indeterminate number whose probable range is known, but which has unknown position within its boundaries. The GN's are defined with the help of the real intervals. More explicitly, if $\boldsymbol{R}$ denotes the set of real numbers, a GN, say A, can be expressed mathematically by

$$A \in [a, b] = \{x \in \boldsymbol{R}: a \leq x \leq b\}.$$

If $a = b$, then A is called a *white number* and if $A \in (-\infty, +\infty)$, then it is called a *black number*. Compared with the interval [a, b] the GN A may enrich its uncertainty representation with a *whitenization function*, defining a *degree of greyness* for each x in $[a, b]$. For general facts on GN's we refer to Liu and Lin [17].

The well known arithmetic of the real intervals (Moore et al. [18]) is used to define the basic arithmetic operations among the GN's. More explicitly, if $A \in [a_1, a_2]$ and $B \in [b_1, b_2]$ are given GN's and k is a positive real number, one defines:

- *Addition* by $A + B \in [a_1 + b_1, a_2 + b_2]$
- *Subtraction* by: $A - B = A + (-B) \in [a_1 - b_2, a_2 - b_1]$, where $-B \in [-b_2, -b_1]$ is defined to be the *opposite* of B.
- *Multiplication* by: $A \times B \in [min\{a_1b_1, a_1b_2, a_2b_1, a_2b_2\}, max\{a_1b_1, a_1b_2, a_2b_1, a_2b_2\}]$
- *Division* by: $A : B = A \times B^{-1} \in [min\{\frac{a_1}{b_1}, \frac{a_1}{b_2}, \frac{a_2}{b_1}, \frac{a_2}{b_2}\}, max\{\frac{a_1}{b_1}, \frac{a_1}{b_2}, \frac{a_2}{b_1}, \frac{a_2}{b_2}\}]$, where $0 \notin [b_1, b_2]$ and $B^{-1} \in [\frac{1}{b_2}, \frac{1}{b_1}]$ is defined to be the *inverse* of B .
- *Scalar multiplication* by: $kA \in [ka_1, ka_2]$, where k is a positive real number.

Observe that $B + (-B) \in [b_1 - b_2, b_2 - b_1] \neq [0, 0] = 0$,

$B + (-B) \neq (-B) + B \neq 0$ and $B \times B^{-1} = B^{-1} \times B \in [\frac{b_1}{b_2}, \frac{b_2}{b_1}] \neq [1, 1] = 1$.

The white number with the highest probability to be the representative real value of the GN $A \in [a, b]$ is denoted by w(A). The technique of determining the value of w(A) is called *whitenization* of A. If no whitenization function is assigned to A, one usually takes

$$w(A) = \frac{a+b}{2}. \quad (1)$$

4. The Assessment Method Using Grey Numbers

Assume that one wants to evaluate the mean performance of group of n students with respect to the qualitative grades A = excellent, B = very good, C = good, D = fair and F = unsatisfactory (failed). For this, a numerical scale of scores from 1 – 100 is assigned to those grades as follows: A (85 - 100), B (75 - 84), C (60 – 74), D (50 - 59) and F (0 - 49)[1].

[1] One could add more qualitative grades for the assessment, e.g. E = less than satisfactory, could be added between D and F, etc. Also the assignment of the numerical scores to each grade is not uniquely made, depending on the user's personal goals. For example, in a more strict assessment one could take A (90-100), B(80-89), C (70–79), D (60-69),
F (0-59), etc. However, the above alternatives do not change the generality of the assessment method proposed.

We correspond now to each grade a GN, denoted for simplicity with the same letter, as follows:

A∈ [85, 100], B∈[75, 84], C∈ [60, 74], D∈[50, 59] and F∈[0, 49]. Let us denote by n_A, n_B, n_C, n_D and n_F the numbers of students who earned the grades A, B, C, D, F respectively. Assigning to each student the GN corresponding to his (her) individual performance we define the *mean value* of all those GN's to be the GN

$$M = \frac{1}{n}\,[n_A A + n_B B + n_C C + n_D D + n_F F] \qquad (2)$$

Since $n_A A \in [85n_A, 100n_A]$, $n_B B \in [75n_B, 84n_B]$, $n_C C \in [60n_C, 74n_C]$, $n_D D \in [50n_D, 59n_D]$ and $n_F F \in [0n_F, 49n_F]$, it turns out that $M \in [m_1, m_2]$, with

$$m_1 = \frac{85n_A + 75n_B + 60n_C + 50n_D + 0n_F}{n}, \quad m_2 = \frac{100n_A + 84n_B + 74n_C + 59n_D + 49n_F}{n} \qquad (3)$$

Furthermore, since no whitenization functions have been assigned to the GNs A, B, C, D and F and therefore to M also, one may take

$$w(M) = \frac{m_1 + m_2}{2} \qquad (4)$$

Observe that in the extreme case where the maximal possible numerical score corresponds to each student for each grade, i.e. the n_A scores corresponding to A are 100, the n_B scores corresponding to B are 84, etc., the mean value of all those scores is equal to m_2. Also, in the opposite extreme case, where the minimal possible numerical score corresponds to each student for each linguistic grade, i.e. the n_A scores corresponding to A are 85, the n_B scores corresponding to B are 75, etc. the mean value of all those scores is equal m_1. Consequently the value of w(M) gives a reliable approximation of the student mean performance and therefore it is useful when no numerical scores but only qualitative grades are used for the student assessment.

5. The Classroom Experiment on Flipped Learning

The subjects of the classroom experiment were the students of the first term of studies of two engineering departments of the School of Technological Applications of the Graduate T. E. I. of Western Greece. The curriculum of both departments includes a common introductory course of mathematics involving Differential and Integral Calculus in one variable and elements from Analytic Geometry and Linear Algebra. The course was taught for both departments by the same instructor and the students used the same educational material (books, lecture notes, etc.).

The experiment took place during the teaching of the Conic Sections. Five one-hour lectures were devoted on the subject for both departments for teaching the circle, the ellipse, the parabola, the hyperbola and the general equation of the Conic Sections respectively. Note that, the students of the two departments had the same mathematical background from their school education and their grades obtained in the general examination of mathematics for their entry in the tertiary education proved that they had almost the same (in average for each department) mathematical skills.

A traditional teaching methodology was followed for the first department (D_1), which played the role of the *control group*. The instructor developed step-by-step the subject on the board having in mind to keep a dialogue with the students by addressing suitable questions.

The theoretical instruction was followed by a series of exercises, applications and problems, where students took an active part. The purpose of this treatment was to embed the acquired theoretical knowledge, to generalize it to a variety of situations and finally to categorize it in the student cognitive schemas, so that to be able to retrieve it from their memory whenever is needed and to use it properly for tackling related problems.

On the contrary, the flipped teaching methodology was applied for the students of the second department (D_2), which will be referred here as the *experimental group*. In applying that methodology the instructor addressed the new knowledge for each of the five lectures on the subject in the form of a video presentation, which was delivered each time to the students in a cd form at the end of the previous lecture. Those video presentations were designed in cooperation of the instructor of the course with the author of the present paper who conducted the research and the Mathcad mathematical software was used in them for drawing the graphs of the Conic Sections. In the classroom the students were divided to small collaborating groups and they were left to work alone on their papers for solving the given by the instructor exercises and problems connected to the new knowledge. The instructor was inspecting their efforts and guided their inquiries with proper hints or instructions. At the next stage the student groups announced their solutions or efforts to solve the given exercises and problems. A guided by the instructor discussion followed analyzing the reasons of possible failures and comparing the solutions obtained. The last stage involved drawing the final conclusions and the rigorous formation of the new knowledge and concepts. .

One week after the end of the teaching process a written test was performed for both departments involving theoretical questions and problems (see Appendix 1). The individual student performance in the test was assessed with the help of the linguistic grades A, B, C, D and F. Table 1 depicts the performance of the two departments in the test:

Table 1: The outcomes of the test

Grade	**D_1**	**D_2**
A	3	19
B	10	9
C	20	19
D	34	15
F	8	10
Total	**75**	**72**

The GPA index (Voskoglou [12], Chapter 6, p.125) is calculated by the formula:

$$\text{GPA} = \frac{0n_F + 1n_D + 2n_C + 3n_B + 4n_A}{n} \qquad (5)$$

Obviously the GPA index takes values between 0 (worst performance, $n_F = n$) and 4 (best performance, $n_A = n$). Replacing the data of Table 1 to equation (5) one finds that GPA ; 1.55 for D_1 and GPA ; 2.17 for D_2. Therefore the experimental group demonstrated a better quality performance than the control group. More precisely, the difference 2.17- 1.55 = 0.62 between the GPA values for the two groups corresponds to a 15.5% better performance for the experimental group.

Also, replacing the data of Table 1 to equations (3) one finds the values m_1 ; 54.23, m_2 ; 69.29 for D_1 and m_1 ; 58.06, m_2 ; 75.51 for D_2 respectively. Therefore, equation (4) gives that w(M) ; 61.76 for D_1 and w(M) ; 66.78 for D_2 , which shows again that the experimental group demonstrated a better mean performance than the control group. However, in this case the difference 66.78 – 61.76 corresponds only to a 5.02% better performance for the experimental group.

6. Conclusions

The results of the classroom experiment on flipped teaching performed in the present work underlined the superiority of the experimental with respect to the control group. Taking also into account that the flipped teaching methodology was a new experience for the students of the experimental group, we have a strong indication that this methodology could improve further the student performance. However, since a single experiment is usually not enough to obtain definite conclusions, further experimental research is needed for this purpose.

Another point to be discussed here is the fact that the experimental group demonstrated a 15.5% better quality performance, but only a 5.02% better mean performance with respect to the control group. This indicates that the flipped teaching helped more the good students (higher scores). But what happened with the other students? From Table 1 one observes that the failure percentage was greater for the experimental group ($\frac{10}{72} > \frac{8}{75}$), which means that the flipped methodology rather created more problems to the weak in mathematics students. Therefore, another point for future research is to study how one can improve the situation for those students, may be combining the traditional with the flipped teaching methodology.

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Appendix 1

The Questions of the Test

1. Write down the general equation of second degree in two unknowns and the conditions under which it represents a real ellipse, parabola or hyperbola.
2. Design a parabola and find its equation in a suitable coordinate system.
3. Find the equations of the tangents of the hyperbola $x^2-y^2 = 8$ which form an angle of 60^o with the axis XX΄.
4. Find the locus of the centers of the circles touching inside the circle $x^2+y^2-4x-21 = 0$ and outside the circle $x^2+y^2 = 1$.