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Smart Learning Systems: A Markov Chain Approach

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Abstract

A smart learning system (SLS) is a knowledge-based software acting as an intelligent tutor for teaching and training purposes. A SLS is technically difficult to be built and maintained. In this paper a Markov chain model is introduced on the steps of the construction of a SLS evaluating the difficulties that appear during this process. Examples of SLSs for teaching Mathematics are also presented illustrating the applicability of the model in real situations.

Keywords: *Artificial Intelligence (AI), E-learning, Smart Learning System (SLS), Markov Chain (MC), Absorbing MC (AMC)*.

1. Introduction

The Artificial Intelligence (AI) experts have recently developed through the Internet a new generation of smart learning systems (SLS) acting as an intelligent tutor in various educational situations. As we shall show in the next section, a SLS is technically difficult to be built and maintained. This gives the motivation to search for measures evaluating the difficulties appearing during the process of constructing a SLS. In the present article we introduce a Markov chain (MC) on the steps of the construction of a SLS and with the help of it we obtain a measure of those difficulties.

More explicitly, the rest of the paper is organized as follows: In Section 2 the background information is presented about e-learning and the structure of a SLS, needed for a better understanding of this work. A flow-diagram is also designed of the process of constructing a SLS. With the help of this diagram, an absorbing MC (AMC) is introduced in Section 3 on the steps of the construction of a SLS and through it a measure is obtained of the difficulties appearing during this process. An Examples of SLSs for teaching Mathematics are also presented illustrating the applicability of the model in real situations. The article closes with Section 4, where the final conclusions are presented and a short discussion is performed about the perspectives of future research on the subject

2. Distance Learning and Smart Learning Systems

AI focuses on creating “smart machines” being able to think, hear, talk, walk and even feel, like humans [1, 2]. The term AI was coined by J. McCarthy in 1956 [3]. Since then AI has been rapidly developed, covering today a wide range of research areas and technologies and having the potential to create great benefits for the humanity.

Between 1982 and 1984 several studies, mainly in the US, proved that students receiving individual tutoring performed much better than those attending a common class. Thus, the idea of replacing teachers with computers supplied by the proper software for individual tutoring started to be developed. Nowadays two China’s companies, Squirrel and Alo7, are in the front line of pursuing the AI to tutoring [4]. Therefore, Chinese students, especially in Higher Education, are prepared today for the future education.

Thanks to the progress of technology, the distance learning, usually referred as e-learning [5, 6], can replace today a great part of the traditional teaching methods for educating and training purposes (e.g. staff and partners of great companies, etc.) in a much lower cost (travel, accommodation, books, teachers' salaries and other relative expenses) and time/place flexibility (anywhere and at any time).

A smart learning system (SLS), otherwise termed as an intelligent tutoring system (ITS), is a knowledge-based software acting as an intelligent tutor for teaching and training purposes ([7], Section 4, p.8). The target of a SLS is to create an ideal learning environment in which students feel comfortable to ask questions and share inner thoughts. This increases the interaction of students and teachers during the learning process, improves the student creativity and gives them a better understanding of the concepts.

The successive steps for the construction of a SLS involve:

- **S_1:** Development of the knowledge base.

  For example, the knowledge base of an introductory course in Euclidean Geometry should include the properties of a triangle, of a polygon and of a circle, calculation of areas, Pythagorean Theorem, etc. The choice of the most appropriate amongst the existing techniques for presenting the contents of the knowledge base, e.g. lists, production rules, cases, trees, etc., is crucial for the success of this step.

- **S_2:** Choice of the suitable inference and reasoning methodology.

  For example, qualitative reasoning, model based, geometric, probabilistic, fuzzy reasoning, etc.

- **S_3:** Selection of the proper authoring shells.

  Those intelligent shells (proper software) enable the instructor to enter the knowledge base without requiring programming skills. They also facilitate the presentation and study of the exercises, problems and examples. In addition, the teacher has the possibility to specify the best way to teach a particular student and to choose actions determining the student mastery [8].

A graphical representation of the process of constructing a SLS is shown in Figure 1.

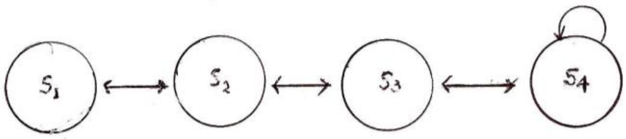

Figure 1: Construction of a SLS

In Figure 1, S_1, S_2 and S_3 denote the successive steps of constructing a SLS that have been described before, while S_4 denotes the final step of the function of the SLS. Note that the first three steps are continuous, since the completion of each one of them usually needs some time characterized by transitions between hierarchically neighbouring steps. For example, being at the step S_2 (reasoning methodology) one may return to S_1 to complete some parts of the knowledge base, being in S_3 (authoring shells) he/she may return to S_2 for making changes or even completely replacing the reasoning methodology, if the initially selected one does not properly fit to the chosen shells, etc.

In conclusion, the development and maintenance of a SLS is a complex and difficult process, requiring a variety of special knowledge and skills from the developing team (AI experts, computer programmers, educators and teachers, etc.). It is therefore very useful to search for measures evaluating the difficulties appearing during the construction of a SLS.

3. The Markov Chain Model

The basic principles of the MC theory were introduced in 1907 by the Russian mathematician A. Markov (1856-1922) through his efforts of coding literal texts. Since then, the corresponding theory has rapidly developed and its importance for probabilistic reasoning has been recognized to the natural, social and applied sciences [9-13].

A MC is defined as a stochastic process that moves in a sequence of steps through a set of states and has only a one-step memory. This means that, being in a certain state at a certain step, the probability of entering another (or the same) state in the next step (transition probability) depends only on the state occupied in previous step and not in earlier steps (Markov property).

However, since the application of the MC theory enables one to make accurate forecasts for the evolution of a great number of random phenomena, many authors, in their effort to cover as many as possible such phenomena using MC's, have weakened the Markov property by accepting that, although the transition probability from one state to another depends mainly on the state occupied in the previous step, need not be completely independent from the states occupied in previous steps (e.g. see Section 12 in Chapter IV of [14], etc.).

When the set of states of a MC is finite, the chain is called a finite MC. For basic facts about a finite MC we refer to Chapters 2 and 3 of the book [13], while for more detailed proofs the reader may look the classical on the subject book [9] or any other of the many modern books that are available in the literature (e.g. [10-12], etc.).

Here, we introduce a finite MC on the steps S_i, i = 1, 2, 3, 4, of the construction of a SLS, as they have been described in the previous section. S_1 is always the starting state of the MC. Further, when the chain reaches the state S_4 it is impossible to leave it (final step). In other words, S_4 is an absorbing state of the MC. Further, since it is possible from any state to reach the absorbing state S_4, not necessarily in one step (see Figure 1), our MC is an AMC with S_4 being its unique absorbing state.

Denote by p_{ij} the transition probability from state S_i to S_j, i, j = 1, 2, 3, 4. Then, with the help of Figure 2, one finds that the transition matrix of the MC is

$$A = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \end{array} \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & S_3 & S_4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ p_{21} & 0 & p_{23} & 0 \\ 0 & p_{32} & 0 & p_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

with $p_{21}+p_{23} = p_{32}+p_{34} = 1$ (probability of the certain event).

Consider the probability vector $P_i = [p_1^{(i)}\ p_2^{(i)}\ p_3^{(i)}\ p_4^{(i)}]$, where $p_k^{(i)}$ denotes the probability of the MC to be in state S_k at the i-th step, k=1, 2, 3, 4 and i = 0, 1, 2,.... .

Then, since S_1 is always the starting state, we have that $P_0 = [1\ 0\ 0\ 0]$. Therefore, according to the general theory of the finite MC's, one finds that

$$\begin{aligned} &P_1 = P_0A = [0\ 1\ 0\ 0] \\ &P_2 = P_1 A = [p_{21}\ 0\ p_{23}\ 0] \qquad (1) \\ &P_3 = P_2 A = [0\ p_{21}+ p_{23}p_{32}\ 0\ p_{23}p_{34}] \\ &P_4 = P_3A = [p_{21}^2+p_{21}p_{23}p_{32}\ 0\ p_{21}p_{23}+ p_{23}^2 p_{32}\ \ p_{23}p_{34}] \text{ and so on.} \end{aligned}$$

Applying now standard results of the theory of the AMC's, we bring the transition matrix A to its standard form A* by listing its absorbing state first and we make a partition of A* to sub-matrices as follows:

$$A^* = \begin{array}{c} \\ S_4 \\ S_1 \\ S_2 \\ S_3 \end{array}\!\!\begin{array}{c} \begin{array}{cccc} S_4 & S_1 & S_2 & S_3 \end{array} \\ \left[\begin{array}{c|ccc} 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 \\ 0 & p_{21} & 0 & p_{23} \\ p_{34} & 0 & p_{32} & 0 \end{array}\right] \end{array}.$$

Therefore, the transition matrix of the non-absorbing states is equal to

$$Q = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \end{array}\!\!\begin{array}{c} \begin{array}{ccc} S_1 & S_2 & S_3 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ p_{21} & 0 & p_{23} \\ 0 & p_{32} & 0 \end{bmatrix} \end{array}.$$

Denote by I_3 the 3X3 unitary matrix. The determinant $D(I_3\text{-}Q) = 1\text{-}p_{23}p_{32}\text{-}p_{21} = p_{23}\text{-}p_{23}p_{32} = p_{23}(1\text{-}p_{32}) = p_{23}p_{34} \neq 0$, therefore I_3-Q is an invertible matrix. Then the fundamental matrix of the AMC is calculated by

$$N = \left(I_3\text{-}Q\right)^{-1} = \frac{1}{p_{23}p_{34}}\begin{bmatrix} 1\text{-}p_{32}p_{23} & 1 & p_{23} \\ p_{21} & 1 & p_{23} \\ p_{21}p_{32} & p_{32} & p_{23} \end{bmatrix} = [n_{ij}],\ i, j = 1, 2, 3.$$

It is well known that the element n_{ij} of N is equal to the mean number of times in state S_j before the absorption, when the MC starts from the state S_i, where S_i and S_j are non-absorbing states. Therefore, since in our case S_1 is always the starting state, the mean number of steps taken before the absorption is equal to

$$t = \sum_{j=1}^{3} n_{1j} = \frac{2+p_{23}(1\text{-}p_{32})}{p_{23}p_{34}} = \frac{2+p_{23}p_{34}}{p_{23}p_{34}} \qquad (2)$$

The minimal value of t is equal to 3, corresponding to the case where we have no backward transitions between neighbouring states (Figure 1). It becomes also evident that the greater is the value of t, the more the difficulties during the process of constructing the SLS. Another indication about those difficulties is of course the total time spent by the designers to build up the SLS.

The following examples of SLSs for teaching mathematics illustrate the applicability of the previous MC model to real situations.

Example 1: During the construction of a SLS for an introductory course of Euclidean Geometry the designers, being at the state S_2 (geometric reasoning), realized that the Euclid's axioms were important for the student better understanding of the subject. Therefore, they returned to S_1 for adding those axioms in the knowledge base of the SLS. Further, being at the state of utilizing the proper authoring shells (S_3), they realized that the pure geometric reasoning did not completely fit with those shells and they returned to S_2 to combine the geometric with qualitative reasoning. No other backward transitions took place during the process of constructing the SLS. It is asked to calculate the value of t and the probability to be in state S_2 at the fourth step of the process of constructing the SLS.

Observe with the help of Figure 1 that, when the MC reaches S_2 for first time, it returns to S_1. However, when it comes back from S_1 to S_2 for second time, then it proceeds to S_3, wherefrom it returns to S_2 to proceed again to S_3. Therefore, we have that $p_{21}=\frac{1}{3}$ and $p_{23}=\frac{2}{3}$. In the same way it is straightforward to check that $p_{32}= p_{34}=\frac{1}{2}$. Replacing those values to equation (2) one finds that t = 7 steps.

Also, the third of equations (1) gives that $p_3=[0\ \frac{2}{3}\ 0\ \frac{1}{3}]$. Therefore, the probability to be in state S_2 at the fourth step of the process of constructing the SLS is equal to $p_2^{(3)} = \frac{2}{3}$.

Example 2: A SLS is under construction for teaching the derivative of a function y=f(x). During the choice of the proper reasoning methodology (state S_2) it was realized that more emphasis should be given to the geometric representation of the derivative. As a result, a backward movement to the knowledge base (state S_1) became necessary in order to underline the fact that the derivative of y=f(x) at a point a of its domain is equal to the slope of the tangent of the graph of y=f(x) at the point (a, f(a)). No other backward movements took place during the construction of the SLS. In this case, with the help of Figure 1 it is straightforward to check that $p_{21}=p_{23}=\frac{1}{2}$, $p_{32}=0$ and $p_{34}=1$. Therefore, equation (2) gives that t= 5 steps.

Example 3: In high-school level real numbers are usually defined in terms of their decimal representations. During the design of a SLS for this purpose it was realized that many of the student difficulties for understanding the irrational numbers are due to their previous incomplete comprehension of the rational numbers. As a result, a backward movement from S_2 to S_1 took place in order to add a repetition about rational numbers. A particular emphasis was given to the fact that fractions and periodic decimals are the same numbers written in a different form.

In a later stage, it was also realized that some more details should be added in the knowledge base of the SLS about transcendental numbers. In fact, this new kind of numbers activates student imagination and increases their interest on the subject by creating a pedagogical atmosphere of mystery and surprise. Cantor proved that the set of transcendental numbers has the power of continuous, in contrast to the set of algebraic numbers which is a denumerable set. This practically means that transcendental are much more than algebraic numbers, but the information that we have bout them is very small related to their multitude.

No other backward movements took place during the construction of the SLS. In this case, with the help of Figure 1 it is straightforward to check that $p_{21}=\frac{2}{3}$, $p_{23}=\frac{1}{3}$, $p_{32}=0$ and $p_{34}=1$. Therefore, equation (2) gives that t= 7 steps.

4. Discussion and Conclusion

The theory of MC's brings together linear algebra and probability theory and has been proved ideal for making short run and long run forecasts for the evolution of real world situations characterized by randomness. The long run forecasts become possible when the corresponding situation can be modelled with the help of an ergodic MC ([13], Chapter 2, pp. 36-38).

In the present article an AMC model has been developed representing the construction of a SLS and evaluating the difficulty of this process. Several other applications of AMC's and ergodic MC's to topics like education, decision-making, case-based reasoning, management, economics, etc. have been presented in earlier works of the author ; e.g. see [13], Chapters 2 and 3 and the relative references contained in it. We plan to focus a great part of our future research on this interesting subject and also on creating fuzzy logic models (e.g. see [13] Chapters 4-8) covering situations of uncertainty due to imprecision, which cannot be usually tackled by methods of probability theory.

References

[1]. Mitchell, M., *Artificial Intelligence: A Guide for Thinking Humans*; Parrar, Straus and Gtraux: NY, USA, 2019.

[2]. Kastranis, A., *Artificial Intelligence for People and Business*, O' Reily Media Inc.: Sebastopol, CA, USA, 2019.

[3]. Moor, J., The Dartmouth college artificial intelligence conference: The next fifty years, *AI Mag.*, 27, 87–91, 2006.

[4]. Yang, X., Accelerated move to AI in China, *ECNU Rev. Educ.*, 2, 347–352, 2019.

[5]. Goyal, S., E-Learning: Future of education, *J. Educ. Learn.*, 6, 239–242, 2012**.**

[6]. Granic, A., Experience with usability evaluation of e-learning systems, *Univ. Access Inf. Soc.*, 7, 209-211, 2008.

[7]. Voskoglou, M.Gr. & Salem, A.-B. M., Benefits and Limitations of the Artificial with Respect to the Traditional Learning of Mathematics, *Mathematics*, 8, 611, 2020.

[8]. Salem A.- B. M. & Parusheva, S., Exploiting the Knowledge Engineering Paradigms for Designing Smart Learning Systems, *Eastern-European Journal of Enterprise Technologies*, 92 (2/2), 38-44, 2018.

[9]. Kemeny, J.G. & Snell, J. L., *Finite Markov Chains*, Springer-Verlag: New York, 1976.

[10]. Domingos, P. & Lowd, D., *Markov Logic: An Interface Layer for Artificial Intelligence*, Morgan & Claypool Publishers: Williston, VT, USA, 2009.

[11]. Zucchini, I.L. MacDonald, I.L. & Langrock, R., *Hidden Markov Models for Time Series*, Second Edition, CRC Press: NY, 2016.

[12]. Davis, M.H.A., *Markov Models and Optimization*, Create Space Independent Publishing Platform (Amazon): Columbia, SC., USA, 2017.

[13]. Voskoglou, M.Gr., *Finite Markov Chain and Fuzzy Logic Assessment Models: Emerging Research and Opportunities*, Create Space Independent Publishing Platform (Amazon): Columbia, SC, USA, 2017.

[14]. Kemeny, J.G., Schleifer, A.J. & Snell, J.L, *Finite Mathematics with Business Applications*, Prentice Hall: London, 1962. on Glasersfeld, E., "Learning as a Constructive Activity", in C. Janvier (Ed), *Problems of representation in the teaching and learning of mathematics*, Lawrence Erlbaum, Hillsdale, N. J., USA, pp. 3-17, 1987.