Soft Sets as Tools for Assessing Human-Machine Performance

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ABSTRACT

A soft set, being a parametrized family of subsets of the universal set of the discourse, is a generalization of the concept of fuzzy set proposed on the purpose of dealing with the uncertainty in a parametric manner. The theory of soft sets has found many and important applications to several sectors of the human activity like decision making, parameter reduction, data clustering and data dealing with incompleteness, medical diagnosis in expert systems, etc. In this paper soft sets are used as tools for constructing an assessment model, which is very useful in cases where the assessment has qualitative rather than quantitative characteristics. The soft set model is applied for student and football player assessment, for evaluating mathematical modelling skills and the performance of case-based reasoning systems. Due to its general texture it could be also applied to a variety of other cases for assessing human and/or machine activities and this is an interesting subject for future research.

Keywords: *Fuzzy logic, soft sets, assessment methods, mathematical modelling (MM), case-based reasoning (CBR).*

1. Introduction

Until the middle of the 1960's probability theory used to be the unique tool in hands of the experts for dealing with the existing in real life situations of uncertainty. Probability, however, based on the principles of the bivalent logic, has been proved sufficient for tackling problems of uncertainty connected only to randomness, but not those connected to imprecision or incomplete information of the given data.

The fuzzy set theory, introduced by Zadeh in 1965 [1], and the connected to it infinite-valued in the interval [0, 1] fuzzy logic [2] gave to scientists the opportunity to model under conditions of uncertainty that are vague or not precisely defined, thus succeeding to mathematically solve problems whose statements are expressed in our natural language. Through fuzzy logic the fuzzy terminology is translated by algorithmic procedures into numerical values, operations are performed upon those values and the outcomes are returned into natural language statements in a reliable manner. For general facts on fuzzy sets, fuzzy logic and the connected to them uncertainty we refer to the chapters 4-7 of the book [3].

Fuzzy systems are considered to be part of the wider class of Soft Computing, also including probabilistic reasoning and neural networks, which are based on the function of biological networks [4]. One may say that neural networks and fuzzy systems try to emulate the operation of the human brain. The former concentrate on the structure of the human mind, e.g. the "hardware", and the latter concentrate on the "software" emulating human reasoning.

A lot of research has been carried out during the last 60 years for improving and extending the fuzzy set theory on the purpose of tackling more effectively the existing uncertainty in problems of science, technology and everyday life. Various generalizations of the concept of fuzzy set and relative theories have been developed like the type-2 fuzzy set, the intuitionistic fuzzy set, the neutrosophic set, the rough set, the grey system theory, etc. [5]. In 1999 Dmtri Molodstov, Professor of the Computing Center of the Russian Academy of Sciences in Moscow, proposed the notion of soft set as a new mathematical tool for dealing with the uncertainty in a parametric manner [6].

Let E be a set of parameters, let A be a subset of E and let f be a mapping of A into the set Δ(U) of all subsets of the universal set of the discourse U. Then the soft set of U connected to A, denoted by (f, A), is defined as the set of the ordered pairs

$$(f, A) = \{(e, f(e)): e \in A\} \quad (1)$$

In other words, a soft set is a paramametrized family of subsets of U. Intuitively, it is "soft" because the boundary of the set depends on the parameters.

For example, let $U=\{H_1, H_2, H_3\}$ be a set of houses and let $E = \{e_1, e_2, e_3\}$ be the set of the parameters e_1=cheap, e_2=expensive and e_3=beautiful. Let us further assume that H_1, H_2 are the cheap and H_2, H_3 are the beautiful houses. Set $A=\{e_1, e_3\}$, then a mapping $f: A \rightarrow \Delta(U)$ is defined by $f(e_1)=\{H_1, H_2\}$, $f(e_3)=\{H_2, H_3\}$. Therefore, the soft set (f, A) representing the cheap and beautiful houses of U is the set of the ordered pairs

$$(f, A) = \{(e_1, \{H_1, H_2\}), (e_3, \{H_2, H_3\})\} \quad (2)$$

A fuzzy set on U with membership function y = m(x) is a soft set on U of the form (f, [0, 1]), where $f(\alpha)=\{x \in U: m(x) \geq \alpha\}$ is the corresponding α – cut of the fuzzy set, for each α in [0. 1]. An important advantage of soft sets is that, by using the set of parameters E, they pass through the existing difficulty of defining properly the membership function of a fuzzy set. For example, defining the fuzzy set of the young people of a country one could consider as young all those being less than 30 years old and another all those being less than 40 years old. As a result they assign different membership degrees to people with ages below those two upper bounds, i.e. they use different membership functions to define the same fuzzy set.

The theory of soft sets has found many and important applications to several sectors of the human activity like decision making, parameter reduction, data clustering and data dealing with incompleteness, etc. [7]. One of the most important steps for the theory of soft sets was to define mappings on soft sets, which was achieved by A. Kharal and B. Ahmad and was applied to the problem of medical diagnosis in medical expert systems [8]. But fuzzy mathematics has also significantly developed at the theoretical level providing important insights even into branches of classical mathematics like algebra, analysis, geometry, topology etc. For example, one can extend the concept of topological space, the most general category of mathematical space, to fuzzy structures and in particular can define soft topological spaces and generalize the concepts of convergence, continuity and compactness within such kind of spaces [9]. The purpose of this article is to present applications of soft sets to assessment processes concerning human and machine activities. A general assessment model using soft sets as tools is developed in the next section and applications of it follow in the third section. The article closes with the final conclusion and some hints for future research on the subject.

2. The Soft Set Assessment Model

Quality is a desirable characteristic of all human activities. This makes assessment one of the most important components of the processes connected to the realization of those activities. The present author has developed in earlier works several methods for assessing human-machine performance under fuzzy conditions, including the measurement of uncertainty in fuzzy systems, the use of the Center of Gravity (COG) defuzzification technique, the use of fuzzy or grey numbers, etc. All these methods have been reviewed in [10]. Here a new model using soft sets is developed for the assessment of human-machine performance in a parametric manner. Such kind of models are very useful when the assessment has qualitative rather than quantitative characteristics

Let U be the set of all objects which are under assessment. Consider the set $E=\{e_1,e_2,e_3,e_4,e_5\}$ of the parameters e_1=excellent, e_2=very good, e_3=good, e_4=mediocre and e_5=failed and the mapping f: $E \rightarrow \Delta(U)$ assigning to each parameter of E the subset of U consisting of all elements of U whose performance is described by this parameter. Then the soft set

$$(f, E) = \{(e_i, f(e_i), i=1, 2, 3, 4, 5\} \quad (3)$$

represents mathematically an assessment of the elements of U in a parametric manner. Note that, without loss of generality and for a more detailed assessment, the set E could include more than five parameters. The following examples illustrate the applicability of this model in real situations.

3. Applications of the model

Example 1: Let $U = \{S_1, S_2, \ldots, S_{30}\}$ be the set of the 30 students of a class. Assume that the first four of them are excellent students, the next eight very good, the following 10 good, the next five mediocre and the rest of them weak students. Let f be the map corresponding to each parameter of E the subset of students of the class whose performance was assessed by this parameter. Then, the soft set

$$(f, E) = \{(e_1, \{S_1, S_2, S_3, S_4\}), (e_2, \{S_5, S_6,\ldots, S_{12}\}), (e_3, \{S_{13}, S_{14}, \ldots, S_{22}\}), (e_4, \{S_{23}, S_{24},\ldots, S_{27}\}), (e_5, \{S_{28}, S_{29}, S_{30}\}) \quad (4)$$

represents in a symbolic way the general performance of the class.

Let also $V= \{C_1, C_2,\ldots, C_{10}\}$ be the set of the different courses taught in the class. Define a mapping g: $E \rightarrow \Delta(V)$ assigning to each parameter of E and for each student of U the subset of V consisting of the courses for which the performance of the student was assessed by this parameter Then the profile of each student can be represented by the soft set

$$(g, E) = \{(e, g(e): e \in E\} \quad (5)$$

For example the soft set

$$(g, E) = \{(e_1, \{C_1, C_8\}), (e_2, \{C_2, C_3, C_5, C_9\}), (e_3, \{C_4, C_6, C_{10}\}), (e_4, \{C_7\}), (e_5, \emptyset)\} \quad (6)$$

represents the profile of a student who demonstrated excellent performance in courses C_1 and C_8, very good performance in courses C_2, C_3, C_5 and C_9, good performance in courses C_4, C_6 and C_{10} and mediocre performance in course C_7.

Example 2: Mathematical modelling (MM) is a very important component of mathematics education, because by applying the mathematical theories to practical needs of our everyday life increases the student interest for mathematics. The main steps of the MM process include [11]:

- S_1: Analysis of the given real world problem.
- S_2: Formulation of the problem and construction of the mathematical model (mathematization).
- S_3: Solution of the model.
- S_4: Validation (control) of the model.
- S_5: Interpretation of the final mathematical results and implementation of them to the real problem.

Mathematization possesses the greatest difficulty among the steps of the mathematical modelling process, because it involves a deep abstracting process, which is not always easy to be achieved by a non-expert. It is sometimes, however, the transition from the solution of the model to the real world (validation and/or implementation of the model) that presents difficulties for students too. Examples illustrating this difficulty are presented in [11].

The use of soft sets enables the representation of the performance of each student in each step of the MM process. In fact, let $V = \{S_1, S_2, S_3, S_4, S_5\}$ be the set of the previously mentioned steps of the MM process. Consider a particular student of the class and define a map $f: E \rightarrow \Delta(V)$ by assigning to each parameter of E the subset of V consisting of the steps of the MM problem assessed by this parameter with respect to the chosen student. For example, the soft set

$$(f, E) = \{(e_1, \{S_1, S_3\}), (e_2, \{S_5\}), (e_3, \{S4\}), (e_4, \{S2\}), (e_5, \emptyset)\} \quad (7)$$

represents the profile of a student who demonstrated excellent performance at the steps of analysis of the problem and solution of the model, very good performance at the step of interpretation of the mathematical results, good performance at the step of validation of the model and mediocre performance at the step of mathematization (he/she faced difficulties, but he/she finally came through).

Example 3: The coach of a football club wants to assess the following characteristics of his players: D=dribbling, P=passing, F= foot kick (shoot), H=head kick, C=creativity and S=speed. Set $U = \{D, P, F, H, C, S\}$ and define a mapping $f: E \rightarrow \Delta(U)$ assigning to each parameter of E and for each player of the club the subset of U consisting of the player's characteristics assessed by this parameter. In this way the coach can represent each player's profile with the help of a soft set. For example, the soft set

$$(f, E) = \{(e_1, \{P, C\}), (e_2, \{F\}), (e_3, \{D\}), (e_4, \{S\}), (e_5, \{H\})\} \quad (8)$$

represents the profile of a player with excellent passing and creativity, very good shoot, good dribbling, mediocre speed, but not good head kick.

In an analogous way one can express the general players' performance with respect to each characteristic of U. Consider for example dribbling (D) and let $V=\{P_1,P_2,\ldots,P_{20}\}$ be the set of all players. Define a map $g: E \rightarrow \Delta(V)$ assigning to each parameter of E the subset of V consisting of the players whose dribbling was assessed by this parameter. Then, the general players' performance with respect to dribbling is expressed by a soft set of the form (3). We could have, for example, that

$$(g, E) = \{(e_1,\{P_1,P_2,P_3\}), (e_2, \{P_4,P_5,\ldots,P_{10},\}), (e_3, \{P_{11}, P_{12},\ldots, P_{15}\}), (e_4, \{P_{16}, P_{17}, P_{18}\}), (e_5, \{ P_{19}, P_{20}\})\} \quad (9)$$

This means that the first three players have excellent dribbling, the next seven very good, the next five good, the next three mediocre and the last two players have no good dribbling.

Example 4: Case-Based Reasoning (CBR) is the process of solving problems based on the solutions of previously solved analogous problems [12]. The use of computers enables the CBR systems to preserve a continuously increasing "library" of previously solved problems, referred as past cases, and to retrieve each time the suitable one for solving a given new problem. The CBR process involves the following steps:

- Retrieve (R_1) the most similar to the new problem past case, or cases.
- Reuse (R_2) the information and knowledge in that case to solve the new problem.
- Revise (R_3) the proposed solution.
- Retain (R_4) the parts of this experience likely to be useful for future problem-solving.

The quality of a CBR system can be assessed with the help of soft sets as follows:

Set $U=\{R_1, R_2, R_3, R_4\}$ and define a mapping f: $E \to \Delta(U)$ assigning to each parameter of E the subset of U consisting of the CBR steps whose quality was assessed by this parameter. For example, the soft set

$$(f, E) = \{(e_1,\{R_1, R_4\}), (e_2, \{R_2\}), (e_3, \{R_3\}), (e_4, \emptyset), (e_5, \emptyset)\} \quad (10)$$

corresponds to a CBR system which demonstrated excellent performance at the steps of retrieval and retaining of the past cases, very good performance at the step of reusing them and good performance in revising the selected past case for obtaining the solution of the new problem.

Also, given a set of CBR systems, one can compare their performance, similarly to the previous examples, with respect to each of the steps of the CBR process.

4. Discussions and Conclusion

From all those exposed in this study it turns that soft sets offer a potential tool for a qualitative assessment of human-machine performance in a parametric manner. Examples were presented to illustrate the applicability of the soft set assessment model to real situations.

Due to its general texture this model could be also applied to a variety of other cases for assessing human and/or machine activities and this is an interesting subject for future research. Another interesting subject for future research could be the development of alternative assessment models under fuzzy conditions by using other types of generalizations of fuzzy sets or related theories [5].

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