Neutrosophic Sets as Tools in Assessment Processes

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Abstract

The concept of neutrosophic set, introduced by Smarandache in 1995, added the degree of indeterminacy or neutrality to Atanassov's intuitionistic fuzzy set, thus being a wider generalization of fuzzy sets. In this work an innovative assessment method is presented using neutosophic sets as tools and it is compared with an earlier author's assessment method using closed real intervals (grey numbers) as tools. The new method is very useful when the person who makes the assessment is not completely sure about the exact characterization of the individual performance of some (or all) of the objects under assessment.

Keywords: Fuzzy set (FS), neutrosophic set (NS), grey number (GN), assessment under fuzzy conditions.

1. Introduction

The assessment of human or machine activities is a very important process, because it helps to correct mistakes and improve performance. Assessment takes place in two ways, either with the help of numerical or with the help of qualitative grades. The second way is usually preferred when more elasticity is desirable (as it frequently happens, for example, in case of student assessment), or when no exact numerical data are available.

In case of using numerical grades, the calculation of the mean value of all the individual scores is a standard method applied for the overall evaluation of the performance of a group of individuals with respect to a certain activity. This method, however, is not applicable in cases of using qualitative grades, where methods of fuzzy logic are usually preferred. In [1] several methods of such kind are reviewed, developed by the present author in earlier works.

In this paper a new assessment method is developed with the help of *neutrosophic sets (NSs)*. This method, which is compared with an author's earlier assessment method using *closed real intervals (grey numbers)* as tools [1], is very useful when the person who makes the assessment (e.g. a teacher) is not completely sure on how to characterize the individual performance of some (or all) the objects under assessment (e.g. students).

The rest of this work is organized as follows: Section 2 contains the mathematical background about grey numbers (GNs) and NSs, which is necessary for the understanding of the paper. The assessment methods with GNs and NSs are developed in Section 3 with suitable examples illustrating them. The article closes with the final conclusions and a brief discussion about future research presented in Section 4.

2. Mathematical Background

2.1 Closed Real Intervals as Tools in Grey Systems

The theory of *grey systems* (Deng [2], 1982) is an alternative way for managing the uncertainty in case of approximate data. Closed real intervals, are used for performing the necessary calculations in grey systems.

A closed real interval [x, y] can be considered as representing a real number T, called a *grey number (GN)*, with unknown value in [x, y]. We write then $T \in [x, y]$. A GN T is frequently accompanied by a *whitenization function* $f: [x, y] \to [0, 1]$, such that, if f(a) approaches 1, then a in [x, y] approaches the unknown value of T. If no whitenization function is defined, it is logical to consider as a crisp approximation of the unknown number A the real number

$$V(A) = \frac{x+y}{2} \quad (1)$$

The arithmetic operations on GNs are defined with the help of the known arithmetic of the real intervals [3]. In this work we'll only make use of the addition of GNs and of the scalar multiplication of a GN with a positive number, which are defined as follows:

Let $A \in [x_1, y_1]$, $B \in [x_2, y_2]$ be two GNs and let k be a positive number. Then:

- The sum: A+B is the GN $A+B \in [x_1+y_1, x_2+y_2]$ (2)
- The scalar product kA is the GN $kA \in [kx_1, ky_1]$ (3)

We close this subsection with the following definition:

Definition 1: Let $I_1, I_2, \ldots, I_k$ be a finite number of GNs and assume that I_i appears n_i times in an application, i = 1,2,…., k. Set $n = n_1+n_2+\ldots+n_k$. Then the *mean value* of all these GNs is defined to be the GN

$$I = \frac{1}{n}(n_1I_1+n_2I_2+\ldots+n_kI_k) \quad (4)$$

2.2 Neutrosophic Sets

Zadeh, in order to deal with partial truths, defined in 1965 the concept of *fuzzy set (FS)* as follows [4]:

Definition 2: A FS A in the universe U is of the form:

$$A = \{(x, m(x)): x \in U\} \quad (5)$$

In equation (5) $m: U \to [0,1]$ is its *membership function*. The value m(x) is called the *membership degree* of x in A**.** The nearer m(x) to 1, the better x satisfies the characteristic property of A. A crisp subset A of U is a FS on U with its characteristic function being its membership function. Whereas probability theory is suitable for tackling only the uncertainty due to *randomness* (e.g. games of chance), FSs tackle successfully the uncertainty due to *vagueness*, created when one is unable to clearly differentiate between two situations, such as "a good student" and "a mediocre student". For more details on FSs we refer to [5].

Atanassov in 1986 added to Zadeh's membership degree the *degree of non-membership* and introduced the concept of *intuitionistic fuzzy set (IFS)* as follows [6]:

Definition 3: An IFS A in the universal set U is of the form

$$A = \{(x, m(x), n(x)): x \in U, 0 \leq m(x) + n(x) \leq 1\} \quad (6)$$

In equation (6) m: U $\rightarrow$ [0,1] is the membership function of A and n: U $\rightarrow$ [0,1] is its non-membership function.

One can write m(x) + n(x) + h(x) = 1, where h(x) is the *hesitation* or *uncertainty degree* of x. When h(x) = 0, then the corresponding IFS is an ordinary FS. An IFS promotes the intuitionistic idea, as it incorporates the degree of hesitation.

For example, if A is the IFS of the diligent students of a class and (x, 0.7, 0.2) ∈ A, then there is a 70% belief that student x is diligent, a 20% belief that x is not diligent, and a 10% hesitation to be characterized as either diligent or not.

IFSs, simulate successfully the existing *imprecision* in human thinking.

Smarandache, motivated by the various neutralities appearing in real life - like <friend, neutral, enemy>, <small, medium, high>, <win, draw, defeat>, etc. - introduced in 1995 the degree of *indeterminacy/neutrality* of the elements of the universe U and defined the concept of NS the simplest form of which is the following [7]:

Definition 4: A *single valued NS (SVNS)* A in U is of the form

$$A = \{(x,T(x),I(x),F(x)): x \in U, T(x),I(x),F(x) \in [0,1], 0 \leq T(x)+I(x)+F(x) \leq 3\} \quad (7)$$

In (7) T(x), I(x), F(x) are the degrees of *truth* (or membership), indeterminacy and *falsity* (or non-membership) of x in A respectively, called the *neutrosophic components* of x. For simplicity, we write A<T, I, F>.

The term “neutrosophy” is a synthesis of the adjective “neutral′ and of the Greek word “sophia” (wisdom) and means “the knowledge of neutral thought”.

For example, let U be the set of the players of a basketball team and let A be the SVNS of the good players of U. Then each player x of U is characterized by a *neutrosophic triplet* (t, i, f) with respect to A, with t, i, f in [0, 1]. For example, x(0.8, 0.1, 0.3) ∈ A means that the coach of the team is 80% sure that x is a good player, but at the same time he has a 10% doubt about it and a 30% belief that x is not a good player. In particular, x(0,1,0)∈ A means that the coach does not know absolutely nothing about x’s affiliation with A.

Indeterminacy is defined to be everything which is between the opposites of truth and falsity [8]. In an IFS the indeterminacy coincides by default to hesitancy, i.e. we have I(x)=1-T(x) – F(x). Also, in a FS is I(x)=0 and F(x) = 1 – T(x), whereas in a crisp set is T(x)=1 (or 0) and F(x)= 0 (or 1). In other words, crisp sets, FSs and IFSs are special cases of SVNSs.

For more details about SVNSs new refer to [9]

If the sum T(x) + I(x) + F(x) of the neutrosophic components of x ∈ U in a SVNS A on U is <1, then we have incomplete information about x, if is equal to 1 we have complete information, and if is greater than 1 we have *paraconsistent* (i.e. contradiction tolerant) information about x. A SVNS may contain simultaneously elements having all the previous types of information.

When T(x) + I(x) + F(x)<1, ∀ x ∈ U, then the corresponding SVNS is usually referred as *picture FS (PiFS)* [10]. In this case 1- T(x)-I(x)-F(x) is called the degree of *refusal membership* of x in A. The PiFSs based models are adequate in situations where we face human opinions involving answers of types yes, abstain, no and refusal. Voting is a representative example of such a situation.

The difference between the *general definition of a NS* and the previously given definition of a SVNS is that in the general definition T(x), I(x) and F(x) may take values in the *non-standard unit interval*]−0, 1+[(including values <0 or >1). The latter, could in fact happen in real situations [7].

NSs, apart from vagueness, manage as well the cases of uncertainty due to *ambiguity* and *inconsistency.* In the former case the existing information leads to several interpretations by different observers, as it happens, for example, with characterizations like "beautiful" and "not beautiful" Inconsistency, on the contrary, appears when two or more pieces of information cannot be true at the same time, which makes the obtainable in this case information to be conflicted. For example, "the probability for the weather to be sunny tomorrow is 95%, but this does not mean that the probability to have some rains is only 5%, because the meteorological conditions could be changed".

3. The Assessment Methods

3.1 The method with Grey Numbers

We illustrate this method, developed in detail in [1], with the following example:

Example 1: The training staff of a tennis club evaluated its 20 players as follows: The first three of them are excellent players, the next five very good, the next six good, the following four mediocre players and the last two demonstrate a non-satisfactory performance. It is asked to estimate the mean level of the player skills.

Solution: Let U = {p_1, p_2,…, p_{20}} be the set of the players of the club and let A=excellent, B=very good, C=good, D=mediocre and F=not satisfactory be the qualitative grades characterizing the individual level of skills of each player. Translating the previous qualitative grades in the numerical scale 0-100 we assign to each qualitative grade a closed real interval (GN), denoted, for simplicity, by the same letter, as follows: A=[85, 100], B=[75, 84], C=[60, 74], D=[50, 59] and F=[49, 0]. Note that this assignment, although it was performed according to generally accepted standards, is not unique depending on the evaluators' personal criteria (more strict or more elastic assessment).

Then, by equation (4) the mean level of the player skills can be estimated with the help of the GN $M = \frac{1}{20}(3A+5B+6C+4D+2F)$. Using equations (2) and (3) it is straightforward to check that $M = \frac{1}{20}[1190, 1498] = [59.5, 74.9]$. Therefore, by equation (1) one finds that V(M)=67.2, which shows that the mean level of the player skills is good (C).

3.2 The New Method with Neutrosophic Sets

For the development of our method, writing the elements of a NS in the form of neutrosophic triplets, we need to define addition in NSs and the scalar product of a neutrosophic triplet with a positive number.

Addition of neutrosophic triplets is equivalent to the union of neutrosophic sets. That is why addition can be defined in many ways, equivalently to the known in the literature neutrosophic union operators [11].

Here, we define addition and scalar product in a way compatible to the corresponding operations with GNs (see Section 2.1) as follows:

Definition 5: Let A be a SVNS, let (t_1, i_1, f_1), (t_2, i_2, f_2) be in A and let k be appositive number. Then:

- The sum $(t_1, i_1, f_1) + (t_2, i_2, f_2) = (t_1+t_2, i_1+i_2, f_1+f_2)$ (8)
- The scalar product $k(t_1, i_1, f_1) = (kt_1, k\,i_1, kf_1)$ (9)

Further, we define the mean value of a finite number of neutrosophic triplets as follows:

Definition 6: Let A be a SVNS and let, (t_2, i_2, f_2), …., (t_k, i_k, f_k) be a finite number of elements of A. Assume that (t_i, i_i, f_i) appears n_i times in an application, i = 1,2,…., k. Set $n = n_1+n_2+….+n_k$. Then the *mean value* of all these elements of A is defined to be the neutrosophic triplet

$$(t_m, i_m, f_m) = \frac{1}{n}[n_1(t_1, i_1, f_1)+n_2(t_2, i_2, f_2)+….+n_k(t_k, i_k, f_k)] \qquad (10)$$

As it will be illustrated by the following example, our assessment method with NSs is based on equation (10).

Example 2: Reconsider Example 1 and assume that the training staff of the tennis club is not sure about the individual assessment of each player. They decide, therefore, to characterize the set of excellent players using neutrosophic triplets as follows: p_1(1, 0, 0), p_2(0.9, 0.1, 0.1), p_3(0.8, 0.2, 0.1), p_4(0.4, 0.5, 0.8), p_5(0.4, 0.5, 0.8), p_6(0.3, 0.7, 0.8), p_7(0.3, 0.7, 0.8), p_8(0.2, 0.8, 0.9), p_9(0.1, 0.9, 0.9), p_{10}(0.1, 0.9, 0.9} and all the other players by (0, 0, 1). This means that the training staff is absolutely sure that p_1 is an excellent player, 90% sure that p_2 is an excellent player too, but they have a 10% doubt about it and there is also a 10% belief that p_2 is not an excellent player, etc. For the last 10 players the training staff is absolutely sure that they cannot be characterized as excellent players. What should be the conclusion about the mean level of the player skills in this case?

Solution: By equation (10) the mean level of the player skills can be estimated by the neutrosophic triplet $\frac{1}{20}$[(1, 0, 0)+(0.9, 0.1, 0.1)+(0.8, 0.2, 0.1)+2(0.4, 0.5, 0.8)+2(0.3, 0.7, 0.8)+(0.2, 0.8, 0.9)+2(0.1, 0.9, 0.9)+10(0, 0, 1)], which by equations (8) and (9) is equal to $\frac{1}{20}$(4.5, 5.3, 16.3) = (0.225, 0.265, 0.815). This means that a random player of the club has a 22.5 % probability to be an excellent plater, however, there exists also a 26.5% doubt about it and an 81.5% probability to be not an excellent player. Obviously this conclusion is characterized by inconsistency.

Remark: The training staff of the club could work in the same way by considering the NSs of the very good, good, mediocre and weak students and get analogous results.

3.3 Importance and Comparison of the Two Assessment Methods

When using qualitative grades for the assessment, the calculation of the mean value of a group's performance is obviously not possible in the standard way. The two assessment methods presented in this work are very important in practice, because they give a solution to this problem.

The assessment method using GNs is appropriate when the evaluator is absolutely sure for the assessment of the individual performance of each of the objects under assessment and gives a creditable approximation of the group's mean performance. The use of NSs, on the contrary, is appropriate when the evaluator has doubts about the individual performance of some (or all) of the objects under assessment. In this case, the information obtained depends on the choice of the corresponding NS (e.g. excellent players, good players, etc.) and it is possible to be characterized by inconsistency (e.g. in Example 2 a random player of the club has a 22.5 % probability to be an excellent player, but at the same timer a 81.5% probability to be not an excellent player).

4. Discussion and Conclusion

Two assessment methods under fuzzy conditions (with qualitative grades) were studied in the present paper, which enable the evaluation of the mean performance of a group of objects with respect to a certain activity, when qualitative grades are used for the assessment. The discussion performed leads to the following important conclusions:

- The use of closed real intervals (GNs) is suitable when the evaluator is absolutely sure about the characterization of the individual performance of each object under assessment and gives a creditable approximation of the corresponding group's mean performance. Obviously, this approach is very useful when the performance of two or more groups must be compared.
- The use of NSs is suitable when the evaluator has doubts about the individual performance of some (or all) objects under assessment. In this case, the information obtained depends on the choice of the corresponding NS (e.g. excellent players, good players, etc.) and it could be characterized by inconsistency.

The results obtained in this paper show that frequently a combination of two or more of the theories developed for dealing with partial truths and the existing in the real world uncertainty gives better results. This happens not only in assessment cases, but also in decision making, for tackling the uncertainty, and possibly in many other human or machine activities. This is, therefore, an interesting area for further research.

References

[1]. Voskoglou, M.Gr., Methods for Assessing Human-Machine Performance under Fuzzy Conditions. *Mathematics*, 7, article 230, 2019.
[2]. Deng, J., Control Problems of Grey Systems, *Systems and Control Letters,* 288-294, 1982.
[3]. Moore, R.A., Kearfort, R.B., Clood, M.G., *Introduction to Interval Analysis,* 2nd ed., SIAM, Philadelphia, PA, USA, 1995.
[4]. Zadeh, L.A. Fuzzy Sets. *Inf. Control*, 8, 338–353, 1965.
[5]. Klir, G.J. and Folger, T.A., *Fuzzy sets, Uncertainty and Information*, Prentice-Hall, London, 1988
[6]. Atanassov, K.T., Intuitionistic Fuzzy Sets, *Fuzzy Sets and Systems*, 20(1), 87-96, 1986.
[7]. Smarandache, F., *Neutrosophy / Neutrosophic Probability, Set, and Logic*, Proquest, Michigan, USA, 1998
[8]. Smarandache, F., Indeterminacy in Neutrosophic Theories and their Applications, *International Journal of Neutrosophic Science*, 15(2), 89-97, 2021.
[9]. Wang, H., Smarandanche, F., Zhang, Y. and Sunderraman, R., Single Valued Neutrosophic Sets, *Review of the Air Force Academy (Brasov)*, 1(16), 10-14, 2010.
[10]. Cuong, B.C., Picture Fuzzy Sets, *Journal of Computer Science and Cybernetics*, 30(4), 409-420, 2014.
[11]. Smarandache, F., Subtraction and Division of Neutrosophic Numbers, *Critical Review*, Vol. XIII, 103-110, 2016.